by keeping constantly in mind that mathematics is a study of relations between variables, or the properties of functions. Each division will then involve a study of a particular class of functions and we may jump from one subject to another only when the discovery of some relation enables us to extend our knowledge of a previous subject.

The first division, being the introduction, brings in the notion of variable and dependence of one variable upon another, shows how numbers arranged on a line (a scale) enable us to exhibit the correspondence between different values. The graph obtained by placing the scales at right angles should appear to the student only as an easy way of exhibiting this correspondence; and the teacher should emphasize the relation between the points on the scales rather than the points on the graph. The language of coördinates and axes is introduced apparently only as an excuse to simplify the terminology. By emphasizing the fact that a curve is an exhibition of the restriction on, or the relation between, the values of the variables, we can get away from the graphic algebra which teaches the plotting of curves and mistakes the means for the end.

The ideas of such an introduction comprise the first chapter of any text on coördinate geometry, and are presented sooner or later in every course on college algebra, and in all the recent texts on trigonometry. But in none of these places can the emphasis be properly placed on the functional relation. Geometry places the emphasis on coördinates and curves; trigonometry introduces the ideas merely to graph the functions and then stops; the algebras use the curves as geometric means of solving equations. Irrespective of what course the student is about to pursue there is no reason why the year should not begin with a thorough explanation of the notion of function.

The second division then begins with the simplest function, the linear. By presenting the line in its slope form we can introduce the notion of the rate of increase of a function and illustrate how a function is determined by its rate of increase and its value at one point. While this idea will not recur until much later there are nevertheless problems to which it can be applied, for example, the coefficient of linear expansion. We consider briefly the functions of the type x^n, and then formulate the general methods of curve tracing, for example, the change in a curve when in its equation x is replaced by kx or by $x - a$. These methods enable us to study the parabola, ellipse, and hyperbola (without the xy term) and incidentally can furnish a review of quadratics and exponents.

The third division considers what I prefer to call the trigonometric functions rather than trigonometry, for the latter term might not, for example, include polar coördinates except as a side-issue, whereas as in studying functions we would assuredly draw their graphs as an aid in finding their properties, and the polar graph is as useful as the cartesian. While in all teaching theory and application are blended, they are separated in the outline to conform with the idea that we seek the properties of functions first and then use these properties to enlarge our ideas as to previous problems. The applications carry us from field to field but we are thoroughly conscious of our travels and have a legitimate

excuse for the journey. It is this consciousness of change and the deliberateness of it which will remove the difficulties pointed out by E. B. Wilson:[1] ". . . most problems . . . are really problems in algebra, in trigonometry, or in analytics, and the training in classification according to these topics is highly valuable;" for we do classify the problem and then focus on it the nature of the functions involved.

The treatment of the fourth division is along similar lines. We are not interested in logarithms alone, but in the logarithmic function. The subject can be well introduced by a problem involving a function whose rate of increase is proportional to itself, for example, that of determining the number of bacteria in a solution. Any approximate solution leads to an exponential function; the correct solution leads to a splendid explanation of the significance of the constant e and unquestionably the very best explanation a student can have. This has always been one of the weak spots in calculus; merely calling e the natural base does not reveal its significance. Logarithms may then be introduced as answering the question of the time required for the bacteria to reach a specified number. After the customary formal treatment of the uses of logarithms we consider the various exponential relations and the use of logarithmic coördinates. The exponential curve, the catenary, and the hyperbolic functions close the division.

To this point we have consistently presented the fundamental functions, x^n, the trigonometric, and the exponential. The fifth division shows again the unity of the subjects by treating the properties common to all the functions. At this stage the student has worked with a sufficient number of functions to profit by classifying them as odd, periodic, continuous, etc. This is also a suitable place for parts of algebra such as the zeros of functions, sets of zeros of two functions, the remainder theorem, and synthetic division. These subjects now enter as a natural step in finding the properties of functions.

The next part needs perhaps more careful attention than the others because we must make sure that the calculus is not introduced at the expense of other subjects and that the work will not need repetition during the following year. That no subject is slighted will be evident before the end of the article. Any method of correlation will, in fact, enable us to treat more subjects during the year because we have at our disposal the time gained through the avoidance of such repetition as is always unavoidable when algebra, trigonometry and analytics are taught in three distinct courses. Each of these courses, for example, must separately introduce the notions of functions and of coördinates; under the present outline this is done thoroughly once. Again, many of the results of analytics can be derived by the shorter methods of calculus, and this in itself is desirable unless we agree to prefer old methods to new ones merely because they are old.

If less than fifteen or even twenty hours are devoted to the calculus we may feel sure that the student will forget all about it before the following year.

[1] A review of C. S. Slichter's *Elementary Mathematical Analysis* in the MONTHLY for February, 1915, page 56.

For a treatment in twenty-five hours the writer suggests a complete development of the technique of differentiation together with such applications as are found in curve tracing (maxima, minima, and inflexion points), maxima and minima of functions, and problems dealing with velocity and rates. No reference should be made to integral calculus except of course the customary exercises in finding $f(x)$ when its derivative is given and even these should be used only to impress on the memory the standard formulæ of differentiation. With this accomplished the sophomore will have a definite background and the time will not be wasted.

In the course outlined by Mr. Griffin the analytic geometry is postponed until the junior year. Unless, however, the student follows mathematics for three years, analytics can be properly used to fill in the remainder of the freshman year. It will be noticed in the present outline that the line, polar coördinates, and the simple conics have been treated before the sixth division is reached. The conics are now defined by their focal definitions ($r_1 + r_2 = 2a$, etc.), the existence of directrices proved, the eccentricities defined and compared. Then the latus rectum is introduced preliminary to the polar equation, $\rho(1 + e\cos\theta) = l$, which holds for all the conics. The change in the equation of a curve under rotations and the analysis of the general equation of the second degree follows. A study of equations involving arbitrary constants leads to the discussion of lines tangent to a conic where, if the teacher so chooses, the methods of calculus may be used instead of handling the discriminant of the quadratic. By using both methods or by treating these same problems in the previous division the student will see how the new theory improves old methods. The discussion of locus problems leads to such other curves as the witch, cissoid, cycloids, conchoids, etc., and the position of these subjects here is advantageous in that the end of the year will be approaching and so these subjects can be treated as briefly or as thoroughly as time permits.

The student in an engineering school, as a preparation for the sophomore year, needs a knowledge of vectors and this subject ends the year's work. As the theory involves always the use of the trigonometric functions, vectors might be studied as one of the applications of these functions. But the present arrangement preserves the unity of the course, inasmuch as the last division then deals with variables whose values can not be represented by points on a line. The position of vectors and complex numbers at the end of the year again offers possibilities for variation in treatment.

The various divisions of this outline constitute the essential parts of "*An Introduction to the Elementary Functions*," a fit title for this course. The fact that such a title is possible and appropriate is a last proof of the unity of the material. Of course a mere title is of slight value in teaching and any outline can fail unless the presentation has coherence, a thing which must be left to the textbook writer. Also, unless the teacher consistently adheres to the notion that every chapter is a study of the relations between variables, properties of functions and the applications of these properties, all unity is lost.

There are three types of students for whom a course as here outlined is

especially suitable. The non-specialist who wishes an introduction to mathematics can profit less by trigonometry or algebra than by an immediate insight into the theory of functions. Unity is to him of more importance than any other element. The second type is the engineering student who during his freshman year needs an introduction to calculus and to vectors, as a preparation for physics and mechanics, rather than most of the work in college algebra. Partial fractions are best considered preliminary to certain integrations; the binomial theorem fits into the study of series; permutations and combinations are only of the most indirect value. The third type is the student who would later specialize in mathematics, and for him the present outline includes everything that he now gets in his freshman courses of trigonometry, algebra and analytics with the exception of permutations, a loss more than counterbalanced by his knowledge of calculus.

THE DUPLICATION PROBLEM.[1]

By JAMES H. WEAVER, West Chester High School.

There have come down to us from the remote past three problems of perennial interest, namely, the duplication problem, the trisection problem, and the quadrature problem. The first of these has for its object the finding of the edge of a cube that is double a given cube, the second the trisection of any angle, and the third the finding of a square equivalent to a given circle. It is the object of this paper to give:

I. A short historical sketch of the duplication problem, calling attention to the various methods of attacking the problem that were made possible by the advancement of mathematics.

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[1] The following authorities may be consulted for a fuller discussion of the problem:

(*a*) *Collections of Pappus*, ed. Hultsch, Berlin, 1876, page 31 and ff. In addition to the approximate solution mentioned in the text, this contains the principal solutions of the Greeks of the Alexandrian School.

(*b*) *Historia Problematis de Cubi Duplicatione*, by N. T. Reimer, Göttingen, 1798, 8vo, 238 pages. A very full account of the history of the problem.

(*c*) *Historia Problematis Cubi Duplicandi*, by C. H. Biering, Copenhagen, 1844, 4to, 64 pages. This was largely stolen from (*b*) according to S. Günther. See Cantor, *Geschichte der Mathematik*, Volume 4, pages 28–9.

(*d*) W. W. R. Ball, *Mathematical Recreations and Essays*, New York, Sixth Edition, 1914, pages 285–291. The discussion here is merely an outline of a few of the most noted solutions with references to the original sources.

(*e*) *Das Delische Problem*, A. Sturm, Linz, 1895–7. 4to, 140 pages. A full discussion of the problem in all its stages of development.

(*f*) Cantor, *Geschichte der Mathematik*, Vol. I, first edition, page 349. We have here an authoritative historical account of some of the solutions of the Greeks.

(*g*) Article by A. Conti, found in the following books: *Questioni Reguardanti le Matematiche Elementari*, Volume 2, Bologna, 1914, pages 185–231, and *Fragen der Elementar Geometrie*, Theil 2, Leipzig, 1907. Both these books were edited by Enriques. This article gives a very good discussion of some approximate solutions and takes up the question of the impossibility of a solution by means of ruler and compasses.

(*h*) *Famous Problems of Elementary Geometry*, F. Klein, translated by Beman and Smith, Boston, 1897. This volume includes a discussion of the impossibility of the problem by means of ruler and compasses. We will refer to the above mentioned accounts by the names of the authors; thus, Sturm, p. . . ., etc.

II. Solutions by means of the conchoid and the cissoid.

III. An approximate solution, recorded by Pappus, illustrating the difficulty under which the Greeks labored when dealing with problems of this type.

I. Historical Sketch.

Just how the duplication problem originated is not known. It probably dates back to the early Pythagoreans (about 530 B.C.) who had succeeded in finding the side of a square that was double a given square, and proving that the sides of the two squares are incommensurable. After this accomplishment it would be the natural thing to attempt to find the edge of a cube that is double a given cube. But tradition has given the origin of the problem a romantic setting. Eutocius (about 480 B.C.)[1] has preserved for us one version of the story which was related by Eratosthenes (276–194 B.C.) in a letter to king Ptolemy. It is as follows:

"It is related that one of the old tragic poets whom Minos had imported says that when Minos wished to erect for his son Glaucus a tomb and noticed that its dimensions were one hundred feet (ἑκατομ πεδος) on all sides he exclaimed: 'You have enclosed too small a space for a royal tomb. Double it, but forget not the beautiful form. Therefore double each edge of the monument.'

"But he had clearly erred. For by doubling the edges, the surface is made four times as great and the volume is increased eight fold. Nevertheless it made the question of how a body could be doubled without changing its form an object of investigation among geometers, and this is called the duplication of the cube. That is, they set up a given cube and sought to double it."

The first real progress in the solution of the problem was made by Hippocrates of Chios (about 420 B.C.). He reduced it to the one of finding two mean proportionals to two given lines, a form in which it has since been stated.[2] He did not however succeed in finding the mean proportionals. This task was left to Archytus of Tarentum who (about 400 B.C.) accomplished it by means of the intersections of solids.[3] Soon after this Menaechmus (about 340 B.C.), probably

[1] Archimedes, *Opera Omnia cum Commentariis Eutocii*, ed. Heiberg, Leipzig, 1880–1, volume 3, page 102 ff.

[2] Hippocrates noted that if two lines a and $2a$ were given and if two mean proportionals could be inserted between them such that

$$a : x = x : y = y : 2a,$$

then $x^2 = ay$ and $y^2 = 2ax$, from which it readily appears that $x^3 = 2a^3$.

[3] The following is an outline of the solution of Archytus. Let AD be the larger of the two given lines. Then on AD as diameter describe a circle and on this circle as base erect a cylinder. From A in the circle draw the chord AB equal to the smaller of the two given lines and extend this chord until it meets the tangent drawn from the point D. Let this point be P. Then let the triangle APD be revolved about AD as axis generating a cone. Then suppose a semicircle drawn on AD as diameter and perpendicular to the circle ADB, and let this semicircle be so moved that A remains fixed and the semicircle remains perpendicular to the circle ABD. It will then generate a solid. The intersection of the three solids just described will determine a point. From this point draw an element of the cylinder, and let this cut the circle ABD in C. Draw AC. It can then be easily proved that $\overline{AC}^3 = 2\overline{AB}^3$.

following the suggestion of Archytus,[1] discovered the conic sections and used them to give two solutions of the duplication problem.[2] Plato (about 340 B.C.) contrary to his usual custom of dealing with such problems produced a mechanical solution.[3] Nicomedes invented for this purpose the conchoid[4] and Diocles at about the same time (second century B.C.) produced the cissoid.[5] All the other solutions given by the Greeks were either mechanical or depended on the conic sections.[6]

After the decline of Greek geometry nothing was done to advance the problem until the sixteenth century A.D. when Stifel (1486–1567) attacked it from the side of number theory.[7] Then Viète in his geometry (1593) pointed out the fact that every cubic or biquadratic that is not otherwise reducible leads to either the duplication or the trisection problem when solved.[8] Shortly after this Claud Richard records (1645) some solutions that he attributes to Christopher Grienberger, a contemporary, and which employ a new curve called the proportionatrix.[9] Other solutions of a geometric character were given by Grégoire de Saint-Vincent (1647), Newton (1642–1727) and Huygens (1625–1695).[10]

But the answer to the question of the impossibility of a solution of the problem by means of straight lines and circles was finally given by Descartes (1596–1650) in his geometry.[11] In Book II of this work Descartes classified problems as did the Greeks, into plane, solid and linear[12] and proved that, algebraically considered, plane problems correspond to equations of the first degree and of the second degree, solid problems to equations of the third degree, and linear problems to equations of the fourth and higher degrees.

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[1] In his solution Archytus made use of a point determined by the intersections of solids, one of which was a cone. The locus of such a point could be considered as a curve on a cone. This would naturally lead to the investigation of the properties of curves on a cone.

[2] The two solutions of Menaechmus consist analytically in finding (1) the intersection of the two parabolas $x^2 = ay$ and $y^2 = 2ax$ other than the one at the origin, and (2) the intersection of the hyperbola $xy = 2a^2$ and the parabola $x^2 = ay$. For a full discussion of these solutions and the development of conic sections at this period see Heath's introduction to his edition of the *Conics of Apollonius*, Cambridge, 1896.

[3] For a description of the mechanical device of Plato see Sturm, page 49 and ff.

[4] Nichomedes also used this curve to trisect an angle, Pappus, page 57.

[5] A description of these curves and the solutions by means of their aid is given in part II of this paper.

[6] For a description of the other solutions of the Greeks see Sturm, pages 17–97.

[7] Stifel used the theory of irrationals given in Book X of the *Elements*. See Sturm, page 113.

[8] See Sturm, pages 125–7 and Ball, page 290.

[9] A proportionatrix is defined as follows: Describe about C as center the semicircle ADB on the diameter AB, then with CB as diameter draw the semicircle CEB on the same side of the diameter as ADB. Then draw any line BED cutting these semicircles in the points E and D. Then on AB take $BF = BE$. Then describe about F as center with a radius FB a circle cutting DB in G. Then the locus of G for different positions of BED is a proportionatrix.

[10] For an outline of these solutions and for references to the original sources, see Ball, page 290.

[11] The geometry appeared in 1637. In 1649 a Latin edition was put out by Van Schooten. A modernized French edition was published at Paris in 1886 under the title La Géométrie de René Descartes.

[12] Plane problems were those that could be solved by straight lines and circles, solid problems those that required conic sections, and linear problems included all others. See Pappus, pages 54 and 270.

Then in the third book Descartes takes up a discussion of the duplication and trisection problems and shows that the solution of all irreducible equations of the third degree leads to the solution of one or the other of these problems. His procedure is as follows: The intersection (x, y) of the parabola $x^2 = ay$ and the circle $x^2 + y^2 = ax + by$ is such that $a : x = x : y = y : b$. Thus the equation $x^3 = a^2b$ is given by the intersection of a parabola and a circle. But the parabola $y^2 = \frac{1}{4}x$ and the circle $x^2 + y^2 - \frac{1}{4}\frac{3}{4}x + 4ay = 0$ intersect in a point which solves the trisection problem. The ordinate of this point of intersection is given by the equation $4y^3 = 3y - a$. Therefore a parabola and a circle will solve an equation ot the third degree, provided the squared term is missing. But since every cubic may be reduced to this form, every cubic may be solved by the intersection of a parabola and a circle, which is the same thing as solving either the duplication or the trisection problem.[1]

II. Solutions of Nicomedes and Diocles.

Solution of Nicomedes.[2] Nicomedes first considers the problem of constructing two mean proportionals between any two lines of given length. He proceeds as follows. Let CD and DA be two straight lines (Fig. 1). Complete the rectangle $ABCD$, bisect the lines AB and BC in the points L and E. Produce

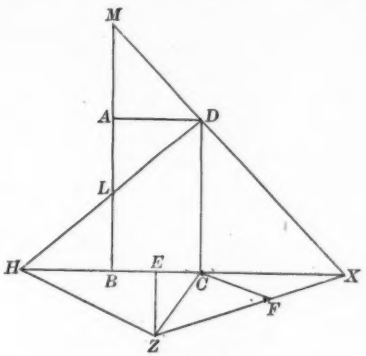

Fig. 1.

the lines DL and CB until they intersect in H. Draw EZ perpendicular to BC so that $ZC = AL$. Then draw ZH and CF parallel to ZH (F being indeterminate). Extend BC to X so that when a straight line is drawn from Z to X, $FX = AL = ZC$. This can be done by means of the conchoid.[3] Then draw

[1] See Sturm, pages 127–131 and Ball, pages 290 and 292.

[2] Pappus, Vol. I, pages 59–63 and pages 249–251.

[3] Z is the fixed point and CF is the fixed line of the conchoid, which is constructed as follows. Let there be a point Z and a line CF each given in position, and a line AL given in magnitude. Then let there be drawn through Z any line as ZX cutting CF in F and let $FX = AL$. The locus of X will be the curve called the conchoid of Nichomedes. There will of course be two branches to the curve, one on each side of CF.

XD and extend it until it meets AB produced in M. Then

$$DC : CX = CX : MA = MA : AD.$$

Proof: Since BC is bisected in E and CX is an extension of BC, then $BX \cdot XC + CE^2 = EX^2$ (Elements, II, 6).

Then by adding EZ^2, $BX \cdot XC + CE^2 + EZ^2 = EX^2 + EZ^2$, or $BX \cdot XC + CZ^2 = ZX^2$. Then by similar triangles, $MA : AB = MD : DX$ and $MD : DX = BC : CX$.

Therefore, $MA : AB = BC : CX$. But $AB = 2AL$ and $BC = \frac{1}{2}HC$, so that $MA : AL = HC : CX$.

Therefore, $MA : AL = ZF : FX$, and by composition $ML : AL = ZX : FX$. But by hypothesis, $AL = FX$, therefore $ML = ZX$, and $ML^2 = ZX^2$. Also (*Elements*, II, 6) $ML^2 = BM \cdot MA + AL^2$, and we have proved above that $ZX^2 = BX \cdot XC + CZ^2$ and $AL^2 = CZ^2$.

Therefore, $BM \cdot MA = BX \cdot XC$, or $BM : BX = CX : MA$.

But on account of parallels MB and DC, $BM : BX = DC : CX$.

Therefore, $DC : CX = CX : MA$.

Also on account of parallels BX and AD, $MB : BX = MA : AD$.

But from above $BM : BX = CX : MA$. Hence $CX : MA = MA : AD$.

Therefore, $DC : CX = CX : MA = MA : AD$. Thus CX and MA are the two required mean proportionals between DC and AD.

From this the duplication problem follows readily, as in note 2, page 107. Because if we have two given lines a and b and if c and d are the two mean proportionals to these lines, then by *Elements, V, Def. 11*; VIII, 12; XI, 33, $a : b = a^3 : c^3$. And if b is double a then c^3 is double a^3.

Solution of Diocles.[1] This solution is different from others because it does not find directly two mean proportionals to two given lines, but, after setting up a cissoid, finds two mean proportionals to two lines that have to each other a ratio equal to the ratio of the two given lines to each other, and from these auxiliary mean proportionals finds those required.

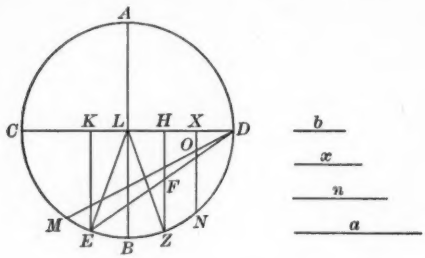

FIG. 2.

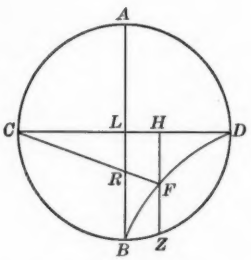

FIG. 3.

Let there be drawn in a circle two diameters perpendicular to each other as AB and CD (Fig. 2) and on either side of B cut off equal arcs EB and BZ, and draw ZH parallel to AB, and draw DE cutting ZH at F. Then ZH and HD will be two mean proportionals between CH and HF.

Proof: Let EK be drawn parallel to AB. Then $EK = ZH$ and $KC = HD$, as is seen if we draw EL and ZL.

For $\angle CLE = \angle ZLD$ and the angles at K and H are right angles. Consequently the triangles ELK and ZLH are congruent, since $LE = LZ$.

[1] This is on the authority of Eutocius, Archimedes, *Opera*, Vol. III, p. 78 ff.

Therefore, $KL = LH$ and hence $CK = HD$.

Since $DK : KE = DH : HF$, on account of similar triangles, and since $DK : KE = KE : KC$, for KE is the mean proportional between DK and KC, then $DK : KE = KE : KC = DH : HF$.

Then, since $DK = CH$, $KE = ZH$ and $KC = HD$, $CH : HZ = HZ : HD = HD : HF$.

This proves that ZH and HD are two mean proportionals between CH and HF.

Again if any two other equal arcs BM and BN are cut off and NX drawn parallel to AB, and DM is drawn cutting NX in O, then by reasoning similar to the above, NX and XD are the two mean proportionals between CX and XO. (The locus of all points determined, such as F and O, is called a cissoid.)[1]

Let there be now two given lines a and b, between which it is desired to find two mean proportionals (Fig. 3). Let there be constructed in a circle the two diameters CD and AB perpendicular to each other, and let the cissoid BFD be described. Let L be the center of the circle and R a point on the radius LB such that $a : b = CL : LR$. Draw CR and produce it until it cuts the cissoid in the point F, and draw through F the line HZ parallel to AB, cutting CD in H and the circle in Z. Then HZ and HD are two mean proportionals between CH and HF. (This has been proved above.)

Since $CH : HF = CL : LR$ and $CL : LR :: a : b$, it is only necessary to insert between a and b two lines, as n and x, so that these four will be in the same ratio as CH, HZ, HD and HF. Then n and x are the two mean proportionals between a and b.[2]

Then, as was shown above in the solution of Nicomedes, $a : b = a^3 : n^3$.

Pappus[3] also gives a solution which is essentially the same as that of Diocles given above. The only difference between the two solutions is this: Pappus, instead of using a curve to determine F, revolves a ruler about D until the portion between CR and RB equals the segment cut off by the line RB and the arc BC. But this is equivalent to laying off equal arcs on either side of AB.

III. Solution of Pappus.

The following is a discussion of an approximate solution proposed to Pappus (end of the third century A.D.) by an unknown geometer. It is recorded in Book III of the *Collections*[4] and is here reproduced with slight modifications of a literal translation. The construction without proof was handed over to Pappus, who shows in his discussion of it, that, although rather long, the construction does nothing more than assume that two mean proportionals are inserted between two given lines (a thing considered impossible by all preceding mathematicians provided only ruler and compasses are allowed in the construction) and that, considered geometrically, it is inexact. Pappus did not seem to realize the fact that the construction affords a method for obtaining an approximate solution of the problem in question. The construction and criticism proceed as follows:

Proposed Construction. Let there be two straight lines AB and AC perpendicular to each other (Fig. 4), and let there be drawn from B a line BD parallel to AC and let $BD = AB$. Join DC and produce DC until it meets AB produced in E, and from E draw EF parallel to BD.

Produce BD, and from D draw a line parallel to BE and intersecting EF in H; and in BD produced take $DN = NL = LX = XK = BD$, and through the points N, L, X, K draw NO, LM, XP, KF intersecting EF in the points O, M, P, F.

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[1] Eutocius however does not give any name to the curve. On the other hand Pappus and Proclus both mention the name cissoid but do not mention the name of the inventor of the curve. Nevertheless, the properties of the curve described by Eutocius seem to coincide with those given by Proclus in connection with the curve which he calls the cissoid. See Sturm, page 87.

[2] The determination of n and x may be made in the following manner. $CH : HZ = a : n$, and $HZ : HD = n : x$, and from this it is evident that $HD : HF = x : b$.

[3] Pappus, pages 64, 167 and 1070.

[4] Pappus, p. 30 and ff.

On KF take a point R so that $KR = AB$, and let KR be bisected in S, and on KF let T and Q be points such that $KF : FS = FS : FT = FT : FQ$.

Then on the line XP take $XX' = AB$, and join $X'K$ and $X'Q$ and draw from the point S a line SY parallel to $X'Q$ and cutting $X'K$ in Y, and from Y the line YW parallel to XK cutting ML in the point W.

Then let there be on ML the points A' and B' such that $LM : MW = MW : MA' = MA' : MB'$. And on ON take $NC' = AB$ and draw $C'L$, $C'B'$, and WD' parallel to $C'B'$ and cutting $C'L$ in D'.

Then from D' draw $D'E'$ parallel to LN and cutting DH in E'. Let Z' and F' be two points on DH such that $DH : HE' = HE' : HZ' = HZ' : HF'$.

Now draw $F'C$ and draw $Z'K'$ and $E'L'$ parallel to $F'C$ and cutting DC in the points K' and L', respectively, and from the points K' and L' draw the lines $K'M'$ and $L'N'$ parallel to AC and intersecting AB in M' and N', respectively.

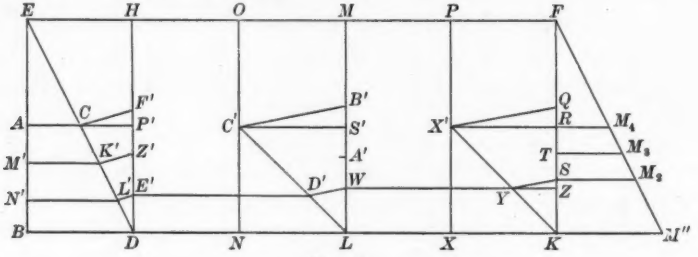

FIG. 4.

Then $M'K'$ and $N'L'$ are two mean proportionals between AC and BD.

Discussion of Pappus. Now the position of the point Q depends upon the ratio $KF : FR = BE : EA$ and hence Q may fall between F and R or between R and T, as we shall show.[1]

First let the ratio be double, that is, $KF : FR = 2 : 1 = 4 : 2$. Then $KF : FS = 4 : 3$ (for by construction $RS = \frac{1}{2}RK$), and from hypothesis $FS : FT = KF : FS$, that is, $KF : FS = FS : FT$ or $4 : 3 = 3 : 2\frac{1}{4}$. But $FS : FT = FT : FQ$, that is, $3 : 2\frac{1}{4} = 2\frac{1}{4} :$ a number less than 2. If, however, we take $FR = 2$, then $FQ < FR$, and Q falls between the points F and R.

Again, let the ratio be quadruple, then $KF : FR = 8 : 2$, and so $KF : FS = 8 : 5$. For, if we suppose that KF contains 8 units and FR 2 units, then $RK = 6$ units and $RS = 3$ units, and, therefore, $FR + RS = FS = 5$ units. But $8 : 5 = 5 : 3\frac{1}{8}$, and $5 : 3\frac{1}{8} = 3\frac{1}{8} :$ a number less than 2, so that, as above, Q will fall between F and R.

Finally, let the ratio be quintuple, that is, $KF : FR = 10 : 2$. Then $KF : FS = 10 : 6$. And $10 : 6 = 6 : 3\frac{3}{5}$, and $6 : 3\frac{3}{5} = 3\frac{3}{5} :$ number greater than 2. Therefore Q falls between the points R and T. (For $FQ > FR$. But the same $FQ < FT$, since by hypothesis $KF : FS = FT : FQ$. Therefore Q falls between R and T.)

Now, since we have proved that Q may fall between F and R or between R and T, we will consider first that, no matter where Q may be assumed, $SF : FT = FT : FR$ is not possible, for that is assuming the thing sought. For let the line XK be produced to M'' so that $KM'' = XK$ and let $M''F$ be drawn and lines parallel to KM'' through the points S, T, and R intersecting FM'' in the points M_2, M_3, M_4, and the problem will be completed as is manifest. Because $KM'' : SM_2 = SM_2 : TM_3 = TM_3 : RM_4$, and $KM'' = BD$ and $RM_4 = AC$. Therefore, SM_2, and TM_3 are two mean proportionals between BD and AC, a thing which is impossible. For since FK is a straight line and in it is a point R, it is not possible by means of the ratios of plane figures to assume two points T and S in FK such that $KF : FS = FS : FT = FT : FR$; for the problem is by nature solid.[2]

Therefore, the proposer did not dare to say that the point Q would fall on R, for that would assume the whole problem, but he has chosen Q between F and R and then finished the construc-

[1] This is shown by special numbers. The method is of interest because it shows how the Greeks were forced to attack problems of this nature because of a lack of algebra.

[2] See note 12, p. 108. Pappus here as elsewhere makes the statement that this problem, as well as the trisection problem, is not possible by means of straight lines and circles. It is in no wise probable that the Greeks ever proved this statement. See Pappus, p. 271.

tion arbitrarily, and gradually fallen back into the first difficulty. He has produced a long construction, not in order to deceive his readers, but he has led himself into error, as I shall show after I have considered the problem in a sensible manner.

Now since the ratio $KF : FR$ is given and FK is given (for it is necessary that each be assumed), then FR is given (*cf.* Euclid's work, *The Data*, proposition 2), therefore the remainder KR is given (*Data*, 4). But SR is given because it is half of RK, and also FR is given, therefore FS is given (*Data*, 3). Therefore the ratio $KF : FS$ is given (*Data*, 1). And by hypothesis $KF : FS = FS : FT$, and we have proved that FS is given, therefore FT will be given. And in the same way FQ will be given. Therefore the difference $FR - FQ$ will be given.

Now suppose the point Q to be between F and R. We have proved this to be possible. Then since the difference $FR - FQ = QR$ is given, and the line joining X' and R is given, because it equals XK, the right triangle $QX'R$ is given in species and magnitude.[1] Therefore the angle RQX' is given, and because of the parallels $X'Q$ and YS we have $\angle RQX' = \angle KSY$.

Now let the line WY be produced till it intersects FK in Z. Then the triangle SZY is given in species (*Data*, 40). But it is also given in magnitude.[2] Therefore YZ is given. Therefore YZ, which is parallel to XK, is in the same straight line with WY. Therefore $WL = ZK$ is given (opposite sides of a parallelogram). Then, because $KF = LM$, and $SK > WL$, for $WL = ZK$, $FS < MW$. Also $KF : FS = FS : FT = FT : FQ$ and $LM : MW = MW : MA' = MA' : MB'$. Therefore $FQ < MB'$.[3] Hence, $KF - FQ > LM - MB'$, that is $B'L < QK$.

Then, again, because WL is given (proved above) and LM is given (since it equals KF which was given), MW is given, and from this the ratio $LM : MW$ is given. And by hypothesis $LM : MW = MW : MA'$, and MW is given, therefore MA' is given.

By similar reasoning MB' may be proved to be given, and from this the point B' is given, and may be between the points S' and M or between S' and A' (S' being a point on ML such that $S'L = KR = AB$).

But if the point B' is assumed to fall on S' it is equivalent to assuming the problem. For, again, in the line ML in which there is a given point S', two points A' and B' are assumed such that $LM : MW = MW : MA' = MA' : MS'$, a thing that no one will grant to be possible. Then if $C'S'$ is drawn, as above, the triangle $S'B'C'$ will be given in species and magnitude and finally DE' will be given. And DH is given and DE' is given, therefore HE' is given, and the ratio $DH : HE'$ is given, and $DH : HE' = HE' : HZ' = HZ' : HF'$. Then (if $DP' = KR$) F' may fall between P' and Z' or P' and H, but not on P' for that is equivalent to assuming the whole problem. But wherever F' may be, let it be supposed to be above P'.

Then when $F'C$ is drawn and parallel to $F'C$ the two lines $Z'K'$ and $E'L'$, and through the points K' and L' parallel to AC the two lines $K'M'$ and $L'N'$ are drawn, it is clear that the problem is not solved. For since $F'C$ is not parallel to EH the angle $CF'H$ will be obtuse if F' falls between P' and H, and acute if F' falls between P' and Z'. But if CP' is drawn, the angle at P' is a right angle. Therefore, the problem will be solved only if F' falls on P', so that in the line DH there are two points E' and Z' such that $DH : HE' = HE' : HZ' = HZ' : HP'$. But if this is not granted the problem can not be solved by means of planes.[4] And this will be persuasive to all by means of numbers, if they admit the table of Ptolemy of lines in a circle.[5] In fact it would have been more satisfactory if he, like the rest, had left the solution in doubt instead of producing it in this manner.

Such is the comment of Pappus. It remained for Günther and Pendlebury, in comparatively recent times, to exhibit the construction of the unknown geometer as one of a series of approximations leading to an accurate solution.[6]

[1] A figure is said to be given in species if its angles and the ratios of its sides are given.

[2] For YS is given because $X'Q : YS = KQ : KS$. From which it readily follows that the triangle YZS is given in magnitude.

[3] For proof of this statement see Pappus, p. 51.

[4] By a "solution by planes" is meant a solution by means of straight lines and circles.

[5] This is the table of chords of Ptolemy given in the *Almagest*. This table uses the measure of the chords to represent the measure of the arcs subtended by the chords. The lengths of the chords of certain arcs in a given circle were known, and, using these to start with, the lengths of the chords of the sum or difference of two arcs or of the half arcs were found. For the method of doing this, see Gow, *Short History of Greek Mathematics*, page 294 f.

[6] For a further discussion of this construction see Sturm, page 94 ff.

FRANK ASBURY SHERMAN.

Frank Asbury Sherman, at the time of his death, February 26, 1915, was professor emeritus on the Chandler foundation in Dartmouth College, having retired from active service in 1911, after a term of 40 years of uninterrupted teaching. He was born in 1841 at Knox, Maine, the son of Harvey Hatch and Elizabeth (Daly) Sherman, and was a direct descendant of William Sherman of the original Plymouth colony, who afterwards settled at Marshfield, Mass. Of stalwart form and sturdy character he was a typical son of the Pine Tree state. He received his early education in her common schools. At the breaking out of the Civil War he was fitting himself for college at the East Maine Conference Seminary. Imbued with a patriotic spirit, possessing the strength and ambition of opening manhood, he heard his country's call to arms and enlisted for three years in the fourth Maine volunteer infantry, in which service he was soon followed by his two brothers, Frederick and Augustus. He was severely wounded in the battle of Fredericksburg and later in the battle of the Wilderness, the latter wound causing the loss of his left arm near the shoulder. He quickly acquired the ability to use his remaining arm for all purposes and was wonderfully independent of the help of others in all the work he undertook. He was in active service through nearly the entire war and was mustered out in March, 1865. He at once resumed his work preparatory to entering college and in 1866 entered the Chandler Scientific Department of Dartmouth College, graduating therefrom in 1870 at the head of his class. He particularly enjoyed mathematics and the sciences. His high scholarship in the former and his early experience in teaching, by which means he had worked his way through college, led to his being invited to teach this subject in the Worcester Polytechnic Institute immediately after his graduation. The next year, his alma mater, needing an instructor in mathematics, tendered him a call as associate professor of mathematics and he was promoted to a full professorship the following year, namely, in 1872. This position he held till 1893, when the Chandler Scientific Department was merged into the College and he was made professor of mathematics on the Chandler foundation. The degree of Master of Science was conferred upon him in 1875. While nominally a professor of mathematics during his entire forty years of active service, for the first twenty years or more he was always ready to fill any gap; and taught various allied subjects, such as descriptive geometry, mechanics, physics, and free-hand drawing, always giving long hours in this service with the same conscientious purpose as had actuated his military career. His willingness to work and to do his part was by no means limited to collegiate lines. He will long be remembered by most of the older alumni as the secretary of their association, serving as he did in that capacity for fourteen years. He also generously served the public in various civic capacities. For many years he was a member of the local board of education and in that position designed and largely superintended the construction of their first high-school building. For several years he was a member of the village governing board. He was not a man who sought popularity or public office, but when called upon to serve, he responded quickly and did his work

promptly, carefully and with fidelity, and in most cases without remuneration. His work as a mathematician was more conspicuous in the class room than in the production of mathematical literature. He was an admirable instructor, a strict disciplinarian, though never harsh. Every student who took his courses felt that he had received full reward for his time and efforts. He took a deep personal interest in the work of each of his students and was beloved, respected, and admired by the alumni, as was shown at every succeeding Commencement, and during his last illness and up to the time of his death.

JOHN VOSE HAZEN.

DARTMOUTH COLLEGE.

NEW BOOKS RECEIVED.

A BUDGET OF PARADOXES. By Augustus De Morgan. Reprinted, with the author's additions, from the Athenæum. Second edition. Edited by David Eugene Smith. The Open Court Publishing Co., Chicago, 1915. Volume I, viii+402 pages. Volume II, 387 pages. $3.50 per volume.

FUNCTIONS OF A COMPLEX VARIABLE. By E. J. Townsend. Henry Holt and Co., New York, 1915. vii+378 pages. $4.00.

DIOPHANTINE ANALYSIS. By R. D. Carmichael. Mathematical Monographs, No. 16. John Wiley and Sons, New York, 1915. vi+118 pages. $1.25.

ANALYTIC GEOMETRY. By H. B. Phillips. John Wiley and Sons, New York, 1915. vii+193 pages. $1.50.

ROBERT OF CHESTER'S LATIN TRANSLATION OF THE ALGEBRA OF AL-KHOWARIZMI, WITH AN INTRODUCTION, CRITICAL NOTES AND AN ENGLISH VERSION. By Louis Charles Karpinski. The Macmillan Co., New York, 1915. 164 pages.

THEORY AND APPLICATIONS OF FINITE GROUPS. By G. A. Miller, H. F. Blichfeldt and L. E. Dickson. John Wiley and Sons, New York, 1916. xvii+390 pages. $4.00.

HISTORICAL INTRODUCTION TO MATHEMATICAL LITERATURE. By G. A. Miller. The Macmillan Co., New York, 1916. xiii+302 pages. $1.60.

FUNDAMENTAL CONCEPTIONS OF MODERN MATHEMATICS. Variables and quantities with a discussion of the general conception of functional relation. By Robert P. Richardson and Edward H. Landis. The Open Court Publishing Co., Chicago, 1916. xxi+216 pages. $1.25.

CONTRIBUTIONS TO THE FOUNDING OF THE THEORY OF TRANSFINITE NUMBERS. By George Cantor. Translated and provided with an introduction and notes by Philip E. B. Jourdain. The Open Court Publishing Company, Chicago, 1915. ix+211 pages. $1.25.

ANALYTIC MECHANICS. By John Anthony Miller and Scott Barrett Lilly. D. C. Heath and Co., Boston, 1915. xv+297 pages. $2.00.

THE ELEMENTS OF SURVEYING AND GEODESY. By W. C. Popplewell. Longmans, Green and Co., London, 1915. xi+244 pages. $2.25.

PLANE AND SPHERICAL TRIGONOMETRY WITH TABLES. By George Wentworth and David Eugene Smith. Ginn and Co., Boston, 1915. iv+230+26+104 pages. $1.35.

PLANE AND SOLID GEOMETRY. By Webster Wells and Walter W. Hart. D. C. Heath and Co., Boston, 1916. vi+467 pages.

SCHOOL ALGEBRA. First Course. By H. L. Rietz, A. R. Crathorne, and E. H. Taylor. Henry Holt and Co., New York, 1915. v+271 pages. $1.00.

SCHOOL ALGEBRA. Second Course. By H. L. Rietz, A. R. Crathorne, and E. H. Taylor. Henry Holt and Co., New York, 1915. x+235 pages. $.75.

FIRST YEAR MATHEMATICS FOR SECONDARY SCHOOLS. 4th edition. By Ernst R. Breslich. University of Chicago Press, Chicago, 1915. xxiv+345 pages. $1.00.

A TWENTIETH CENTURY ARITHMETIC. By C. S. Jackson, F. J. W. Whipple, and Lucy Roberts. J. M. Dent and Sons, London, 1915. viii+495 pages.

PLANE GEOMETRY. By John W. Young and Albert J. Schwartz. Henry Holt and Co., New York, 1915. v+223 pages. $1.00.

HOW TO STUDY AND WHAT TO STUDY. By Richard L. Sandwick. D. C. Heath and Co., Boston, 1915. v+170 pages. $.60.

FUNDAMENTAL SOURCES OF EFFICIENCY. By Fletcher Durell. J. B. Lippincott and Co., Phila., 1914. 368 pages. $2.50.

BOOK REVIEWS.

Send all communications to W. H. BUSSEY, University of Minnesota.

Robert of Chester's Latin Translation of the Algebra of Al-Khowarizmi with an Introduction, Critical Notes and an English Version. By LOUIS CHARLES KARPINSKI, University of Michigan. The Macmillan Co., New York, 1915. 164 pages.

This monograph impresses the reader as being an exceedingly thorough study. It exhibits the modern trend toward a more searching and more critical study of historic material. Its scope is comprehensive. It is much more than a reprint of Robert of Chester's Latin translation from the Arabic, accompanied by a translation into English. It contains in outline the development of algebra before the time of Al-Khowarizmi, an account of Al-Khowarizmi's Algebra and Arithmetic, and their bearing upon the development of mathematics. Much biographical detail is gleaned from out-of-the-way sources, pertaining to Al-Khowarizmi, Robert of Chester and other medieval writers who prepared translations of, or were directly influenced by, the algebra of the great Mohammedan. Robert of Chester's Latin translation was made in the twelfth century. The different extant manuscripts are compared and the deviations from the text of Scheybl, which is followed in this edition, are given in foot-notes with a minutia that seems almost excessive. A Latin glossary assists the reader in making out the medieval

Latin. Added interest is secured by photographic reproductions of pages from the three most complete manuscripts of Robert of Chester's translation. Neither time nor expense has been spared in making the monograph a minute, yet attractive study of the earliest translation into Latin of the famous Arabic text. More profoundly than any other work on algebra that was brought out during the twelve centuries intervening between Diophantus and the Italians, Tartaglia and Cardan has that Arabic text influenced the progress of algebra in the Occident.

FLORIAN CAJORI.

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Analytic Geometry. By H. B. PHILLIPS. John Wiley and Sons, New York, 1915. With answers to exercises. vii+197 pages.

The author tells us in his preface that "he has written this text to supply a course that will equip the student for work in calculus and engineering without burdening him with a mass of detail useful only to the student of mathematics for its own sake. . . . If more than the briefest course is given, the best way to spend the time is in working a large number of varied examples based upon the few fundamental principles which occur constantly in practice."

A wise innovation is a brief discussion of the vector, its use illustrated by a few simple problems such as the division of a line in given ratio, the area of a triangle in terms of the coördinates of its vertices, and the location of the center of gravity of a system of weights. But our author straightway abandons this excellent line of procedure and in the sequel has no regard for the directions on any lines other than the coördinate axes. The angle φ "from the positive direction of the x-axis to the line MN," as used here, is the least positive angle that can so be measured. If two lines L_1 and L_2 make with the x-axis the angles φ_1 and φ_2, then the angle β from L_1 to L_2, according to the text (p. 41), is $\varphi_2 - \varphi_1$ in every case. But if $\varphi_1 > \varphi_2$, this angle is obviously $180° + \varphi_2 - \varphi_1$, unless we regard β as then negative, contrary to custom and contrary to our author himself in the very first application he makes of the familiar formula for $\tan \beta$ (Ex. 3, p. 41. The angle from AB to AC is negative according to the definition of β on p. 41.) It seems to me a matter of regret that the teacher in his class room should accept without comment as to general validity a demonstration based merely on the most obvious geometrical construction. That this careless attitude should find place in our textbooks is a matter for serious concern.

In Art. 26 it seems to me that we have the distance from the line $Ax + By + C = 0$ to the point (x_1, y_1) rather than the distance from the point to the line, but since we are told that such a sign must be used with $\pm \sqrt{A^2 + B^2}$ that the result be positive, this is a matter of little moment.

The definitions of fundamental curves and their properties deserve mention. The slope of the line determined by the points (x_1, y_1) and (x_2, y_2) is *by definition* $m = (y_2 - y_1)/(x_2 - x_1)$, the term inclination is not used. As working tools, one uses only two forms for the rectangular equation of the straight line, $y - y_1 = m(x - x_1)$, and $y = mx + b$. The ellipse is obtained through deformation of

the circle, and it is noted that the ratios obtained by dividing the squares of the distances of any point on the ellipse from the axes by the squares of the parallel semi-axes, have a sum equal to 1. The parabola is defined directly as the locus of points the squares of whose distances from one of two perpendicular lines are proportional to their distances from the other, while the hyperbola is the locus of points the product of whose distances from two lines is constant. This makes possible, without any mention of transformation of coördinates, the derivation of the equations of these curves with any arbitrary pair of perpendicular lines as axes, and leads immediately to a simple classification of the types of curves represented by the general equation of the second degree.

Chapter 5 begins with an excellent discussion of the standard methods of sketching the graph for an algebraic equation. Article 40 on the direction of a curve near a point is notable, since it is the nearest approach to mention of a tangent line to be found in the book. Any discussion of tangent lines is left for the calculus. Such topics as diameters, poles and polars, and their ilk, are also absent. There is a brief discussion of trigonometric and exponential curves, and in the section on empirical equations there are examples of each of the four types $y = mx + b$, $y = ax^2 + bx + c$, $y = ax^n$, and $y = ab^x$.

In the chapter on polar coördinates we find the one time familiar definition of a conic as "the locus of a point moving in such a way that its distance from a fixed point is proportional to its distance from a fixed straight line." Its equation in polar coördinates is changed to rectangular coördinates and the ellipse, hyperbola and parabola are discovered as conics. In the section on the intersection of curves, care is taken to point out that a point may lie on a curve although its coördinates (as given) do not satisfy the equation of the curve. The illustrative example is correct if $a = 1$.

A chapter on parametric representation and one on transformation of coördinates close the portion of the book devoted to plane geometry. A single page is given to the discussion of the general equation of the second degree. As noted above, we already have working rules for the determination of the nature of the locus. The concluding three chapters (32 pages) are devoted to the geometry of space of three dimensions. The vector is again discussed and the equations and properties most frequently needed for the line, the plane, the conicoids and simple curves are brought out. The locus problem is introduced early in the text and is brought in again and again, fixing new principles by repeated use. The problem lists are frequent and extended.

The author seems to have put into this text about what he might offer to his own classes and to have added comparatively few topics merely for the sake of making a complete treatment. This should make the book usable from the point of view of the teacher. One who is looking for a drill course on "the few fundamental principles which occur constantly in practice" should consider this book.

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PROBLEMS AND SOLUTIONS.

EDITED BY B. F. FINKEL AND R. P. BAKER.

[Send all Communications to B. F. FINKEL, Springfield, Mo.]

PROBLEMS FOR SOLUTION.

ALGEBRA.

455. Proposed by JOS. B. REYNOLDS, Lehigh University.

Solve for x_n (not in determinant form) the simultaneous equations,

$$\tfrac{1}{2}x_n + 2x_{n-1} + 2x_{n-2} \cdots 2x_4 + 2x_3 + 2x_2 + 2x_1 = g,$$

$$\tfrac{1}{3}^{0}x_n + \tfrac{2}{3}^{2}x_{n-1} + 8x_{n-2} \cdots 8x_4 + 8x_3 + 8x_2 + 8x_1 = 4g,$$

$$\tfrac{1}{3}^{6}x_n + \tfrac{4}{3}^{0}x_{n-1} + \tfrac{5}{3}^{2}x_{n-2} \cdots 18x_4 + 18x_3 + 18x_2 + 18x_1 = 9g,$$

$$\tfrac{2}{3}^{2}x_n + \tfrac{5}{3}^{8}x_{n-1} + \tfrac{8}{3}^{2}x_{n-2} + \tfrac{9}{3}^{4}x_{n-3} \cdots 32x_4 + 32x_3 + 32x_2 + 32x_1 = 16g,$$

$$\tfrac{3}{3}^{8}x_n + \tfrac{7}{3}^{6}x_{n-1} + 1\tfrac{1}{3}^{2}x_{n-2} + 1\tfrac{3}{3}^{6}x_{n-3} + 1\tfrac{4}{3}^{8}x_{n-4} \cdots = 25g,$$

. . . . . .

$$(\tfrac{1}{2} + 2n - 1)x_n + (\tfrac{1}{2} + 6n - 5)x_{n-1} + (\tfrac{1}{2} + 10n - 13)x_{n-2}$$
$$+ (\tfrac{1}{2} + 14n - 25)x_{n-3} \cdots (\tfrac{1}{2} + 2n^2 - 1)x_1 = n^2g.$$

456. Proposed by PAUL CAPRON, U. S. Naval Academy.

If

$$S_{i,n} = \sum_{k=1}^{k=n-i+1} \frac{(i+k-1)!}{(k-1)!},$$

show that $S_{i,n}$ is equal to $1/(i+1)$ times the last term of $S_{i+1,n+1}$; as, for instance, that

$$S_{1,n} = 1 + 2 + \cdots + n = \frac{n}{2}(n+1),$$

that

$$S_{2,n} = 1\cdot 2 + 2\cdot 3 + \cdots + n(n+1) = \tfrac{1}{3}(n-1)n(n+1),$$

etc.

GEOMETRY.

486. Proposed by ARON INGVALE, Brooklyn, N. Y.

Does the following construction trisect an angle? With the vertex, O, of the given angle as center and with a radius R, describe a circle intersecting the sides of the given angle in A and B. With a radius $\frac{1}{4}R$, and center on OA, describe a circle tangent to the other circle at A and cutting the other side of the angle at E. At E draw a tangent to the last circle and produce it to meet the first circle at F. Draw FO. Then is angle BOF one-third of the angle BOA?

REMARK. Though the construction does not, of course, lead to the trisection of an angle in general, yet as a first approximation it is very good. This fact together with the fact that the construction is very simple, and that the proposer's demonstration that it does trisect the angle is very illusive, are the reasons for giving the problem a place in the MONTHLY.

EDITORS.

487. Proposed by H. B. PHILLIPS, Massachusetts Institute of Technology.

If segments from the vertices A and B of a triangle to the opposite sides are of equal length and divide the angles A and B (measured from AB) proportionally, the triangle is isosceles.

488. Proposed by ROGER A. JOHNSON, Western Reserve University.

If triangles are constructed on a given base, having the radii of the incircle and circumcircle in a constant ratio, determine the locus of the vertex. (Necessarily the constant ratio is not greater than $\frac{1}{2}$.)

CALCULUS.

405. Proposed by CLIFFORD N. MILLS, Brookings, S. Dak.

Determine the greatest quadrilateral which can be formed with the four given sides a, b, c, and d taken in order.

406. Proposed by C. N. SCHMALL, New York City.

Given $f(x+h)+f(x-h)=f(x)\cdot f(h)$, determine by Taylor's theorem or otherwise the nature of the function f.

MECHANICS.

324. Proposed by H. S. UHLER, Yale University.

A rigid body of any shape is at rest in a neutral liquid which is also at rest and has an indefinitely great volume. The body is so situated that the free surface of the liquid is tangent to it at its highest point (or points). All the space above the liquid is filled with a neutral, stagnant fluid whose density is not greater than the density of the liquid. Show that the work done in raising (pure translation) the body very slowly until the interface of the two fluids is tangent to it at its lowest point (or points) is expressible by the formula $mgh - gV(\rho_1 h_1 + \rho_2 h_2)$, where $m \equiv$ mass of body, $V \equiv$ volume of body, $\rho_1 \equiv$ mean density of lower medium in the region involved, $\rho_2 \equiv$ density of upper medium, $h_1 \equiv$ distance of center of mass of displaced liquid below the free surface in the initial position of the body, $h_2 \equiv$ elevation of center of mass of displaced fluid above interface in final position of body, and $h = h_1 + h_2$. (Neglect surface-tension, adhesion, cavities in upper portion of body, etc. This problem arose in connection with a question concerning the raising of a dense object from the bottom of a harbor to the deck of a vessel.)

325. Proposed by CLIFFORD N. MILLS, Brookings, S. Dak.

The lever of a testing-machine is c feet long, and is poised on a knife-edge a feet from one end and b feet from the other, and in a horizontal line, above which the beam is symmetrical. The beam is m inches deep at the knife-edge, and tapers uniformly to a depth of n inches at each end; the width of the beam is the same throughout its length. Find the distance of the center of gravity of the beam from the knife-edge.

NUMBER THEORY.

242. Proposed by NORMAN ANNING, Chilliwack, B. C.

Find a function of n which is equal to A_k when $n \equiv k \pmod{p}$, $k = 1, 2, 3, 4, \cdots, p$.

243. Proposed by CLIFFORD N. MILLS, Brookings, S. Dak.

Determine the rational value of x that will render $x^3 + px^2 + qx + r$ a perfect cube. Apply the result to $x^3 - 8x^2 + 12x - 6$.

Below are given problems in Number Theory proposed between January, 1913, and January, 1915, for which no solutions have been received. Ten problems in this subject were proposed during 1915. For some of these, solutions have been received and others are doubtless under consideration by those interested. They are Nos. 227–236. While not neglecting these more recent ones, may we also have coöperation in clearing up the older list?

191. (Incorrectly numbered 187 in the June, 1913, issue.) Proposed by L. E. DICKSON, University of Chicago.

Find an amicable number triple by solving one of the equations (other than the last) in the MONTHLY, March, 1913, page 92. Note that a solution a is to be excluded if not prime to the numbers in the same line.

192. (Incorrectly numbered 188 in the June, 1913, issue.) Proposed by ARTEMAS MARTIN, Washington, D. C.

Find rational values for v, w, and x that will simultaneously satisfy the conditions:

$$(m^2+n^2)(v^2+w^2+x^2)^2 - 4m^2n^2v^2 + m^2n^2(m^2+n^2) = \square, \tag{1}$$

$$(m^2+n^2)(v^2+w^2+x^2)^2 - 4m^2n^2w^2 + m^2n^2(m^2+n^2) = \square, \tag{2}$$

$$(m^2+n^2)(v^2+w^2+x^2)^2 - 4m^2n^2x^2 + m^2n^2(m^2+n^2) = \square, \tag{3}$$

and n being known quantities.

196. (Incorrectly numbered 192 in the September, 1913, issue.) Proposed by CHARLES MACAULEY, Chicago, Ill.

Combinations containing an even number of letters are formed with the letters a, b, c, d, etc. It is required to place the letters in two columns, so that half the letters in every combination are placed in one column and the other letters of the combination in the other column, and so that all the a's stand in the same column; all the b's in the same column; all the c's in the same column, etc.

198. (November, 1913, issue.) Proposed by ARTEMAS MARTIN, Washington, D. C.

Prove that every even number is the sum of two prime numbers.

Note.—This problem has long been known and no proof has ever been given. EDITORS.

202. (December, 1913.) Proposed by A. R. SCHWEITZER, Chicago, Ill.

There exists an infinitude of systems of dyads $\{\alpha\beta\}$ in 7, 9, 11, etc., elements such that each system has the following properties: (1) if $\alpha\beta$ is in the set, then $\beta\alpha$ is not in the set; (2) for each dyad $\alpha\beta$ in the set there exists an element ξ such that $\xi\beta$ and $\alpha\xi$ are also in the set. For example, such a system is,

12,	23,	34,	45,	56,	67,	78,	89,	91
13,	24,	35,	46,	57,	68,	79,	81,	92
14,	25,	36,	47,	58,	69,	71,	82,	93
51,	62,	73,	84,	95,	16,	27,	38,	49

Investigate the existence of

I. A finite set of triads $\{\alpha\beta\gamma\}$ such that (1) if $\alpha\beta\gamma$ is in the set, then $\beta\gamma\alpha$, $\gamma\alpha\beta$ are also in the set but $\beta\alpha\gamma$ is not in the set, (2) for each triad $\alpha\beta\gamma$ in the set there exists an element ξ such that $\xi\beta\gamma$, $\alpha\xi\gamma$, $\alpha\beta\xi$ are also in the set.

II. A finite set of tetrads $\{\alpha\beta\gamma\delta\}$ such that (1) if $\alpha\beta\gamma\delta$ is in the set, then $\beta\gamma\alpha\delta$, $\gamma\alpha\beta\delta$, $\gamma\delta\alpha\beta$ are also in the set but $\beta\alpha\gamma\delta$ is not in the set, (2) for each tetrad $\alpha\beta\gamma\delta$ in the set there exists an element ξ such that $\xi\beta\gamma\delta$, $\alpha\xi\gamma\delta$, $\alpha\beta\xi\delta$, $\alpha\beta\gamma\xi$ are also in the set.

The problem for alternating *n-ads* for $n > 4$ is obvious.

205. (February, 1914.) Proposed by E. T. BELL, University of Washington.

Show that in the usual arithmetical sense the form that follows admits of composition; give the requisite transformations; and indicate how several, if not all, solutions may be found:

$$x_0^2 + nrx_1^2 + mrx_2^2 + mnx_3^2 + mnrx_4^2 + mn^2r^2x_5^2 + nr^2m^2x_6^2 + rm^2n^2x_7^2.$$

208. (March, 1914.) Proposed by E. T. BELL, University of Washington.

If an odd number is perfect it cannot be the sum of two squares.

209. (March, 1914.) Proposed by R. D. CARMICHAEL, University of Illinois.

Prove that the difference of the sixth powers of an integer cannot be the square of an integer.

211. (April, 1914.) Proposed by E. T. BELL, University of Washington.

If an odd perfect number exists, the total number of its divisors is a multiple of 2, but not of 4; or, what is the same thing, an odd perfect number must be of the form $p^{2a-1}n^2$, where p is prime and a is odd.

214. (April, 1914.) Proposed by A. J. KEMPNER, University of Illinois.

Let a be a positive integer $\geqq 2$, and let $T(n)$ denote the number of distinct divisors of the positive integer n, including both 1 and n, so that $T(1) = 1$, $T(2) = 2$, $T(3) = 2$, $T(4) = 3$, $\cdots$. Show that

$$\Sigma_{n=1}^{n=\infty} T(n)/a^n = \Sigma_{n=1}^{n=\infty} 1/(a^n - 1).$$

The special case $a = 10$ gives, as is easily seen:

$$9\sum_{n=1}^{n=\infty} \frac{T(n)}{10^n} = \frac{1}{1} + \frac{1}{11} + \frac{1}{111} + \frac{1}{1111} + \cdots.$$

217. (May, 1914.) Proposed by E. T. BELL, University of Washington.

(i) If r is a prime greater than 2, and $p = 2^r r + 1$ is prime, the only solution, when n is greater than 2, of $x^n - y^n = p$, is $n = 3$, $x = 2$, $y = 1$.

(ii) The only primes that are simultaneously of the forms $4k + 1$ and $3^m - 2^m$ are 1 and 5.

(iii) Generalize (ii).

219. (June, 1914.) Proposed by R. D. CARMICHAEL, University of Illinois.

Determine whether it is possible for a polygon to have the number of its diagonals equal to a perfect fourth power.

221. (September, 1914.) Proposed by T. E. MASON, Purdue University.

Find a number x such that the sum of the divisors of x is a perfect square. [Carmichael, *Theory of Numbers*, page 17.]

222. (October, 1914.) Proposed by A. H. HOLMES, Brunswick, Me.

Find rational values for m and n such that $(m^2 + 1)/m^2 + (n^2 + 1)/n^2$ may be the square of an integer.

223. (October, 1914.) Proposed by T. E. MASON, Purdue University.

Show that

$$\frac{(rst)!}{t!(s!)^t(r!)^{st}}$$

is an integer, r, s, and t being positive integers. Generalize to the case of n integers, r, s, t, u, $\cdots$. [Carmichael, *Theory of Numbers*, page 28.]

SOLUTIONS OF PROBLEMS.

ALGEBRA.

443. Proposed by A. M. KENYON, Purdue University.

If p_r denote the sum of all the r-factor products that can be formed from the first n natural numbers ($p_r = 0$ for $r > n$), and if

$$D_s = \begin{vmatrix} p_1 & 1 & 0 & \cdots & 0 \\ 2p_2 & p_1 & 1 & \cdots & 0 \\ 3p_3 & p_2 & p_1 & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ sp_s & p_{s-1} & p_{s-2} & \cdots & p_1 \end{vmatrix}$$

show that

$$\sum_{i=0}^{k} (-1)^i c_i \binom{k}{i} D_{2k-i} = 0, \qquad k, n = 1, 2, 3, \cdots,$$

where

$$c_i = \frac{2k+1-i}{1+i}$$

when i is even and $(2n+1)$ when i is odd; and $\binom{k}{i}$ is the coefficient of x^i in $(1+x)^k$.

Solution by the Proposer.

The roots of the equation

$$x^n - p_1 x^{n-1} + p_2 x^{n-2} - \cdots + (-1)^n p_n = 0$$

are the natural numbers $1, 2, 3, \cdots n$.

Solving Newton's formulæ[1] for the sums of like powers of the roots, we obtain

$$1^s + 2^s + 3^s + \cdots + n^s = D_s, \qquad n, s = 1, 2, 3, \cdots.$$

A relation between the D's of odd subscript has been published[2] which is equivalent to

$$(1) \qquad \sum_{i=0}^{I(k/2)} \binom{k}{2i+1} D_{2k-1-2i} = 2^{k-1} D_1{}^k, \qquad k, n = 1, 2, 3, \cdots,$$

and the following relation[3] exists among the D's of even subscript,

$$(2) \qquad \sum_{i=0}^{I(k/2)} \frac{2k+1-2i}{1+2i} \binom{k}{2i} D_{2k-2i} = (2n+1) 2^{k-1} D_1{}^k, \qquad k, n = 1, 2, 3, \cdots.$$

These formulæ, in which $I(k/2)$ denotes the integral part of $k/2$, are readily established by induction. Multiplying (1) by $2n+1$ and subtracting the result from (2) we get the formula sought.

444. Proposed by J. E. ROWE, Pennsylvania State College.

Prove that the determinant

$$\begin{vmatrix} \cot A, & \cot B, & \cot C \\ 1, & 1, & 1 \\ \cos^2 A, & \cos^2 B, & \cos^2 C \end{vmatrix} = 0, \quad \text{if} \quad A + B + C = 180°.$$

Solution by S. E. Rasor, Ohio State University.

Transforming trigonometrically and rearranging, the determinant becomes

$$\frac{-1}{4 \sin A \sin B \sin C} \begin{vmatrix} 2 \cos A \sin B \sin C, & 2 \cos B \sin C \sin A, & 2 \cos C \sin A \sin B \\ 1, & 1, & 1 \\ 2 \sin^2 A, & 2 \sin^2 B, & 2 \sin^2 C \end{vmatrix}.$$

By the formula $2 \sin A \sin B \cos C = \sin^2 A + \sin^2 B - \sin^2 C$ when $A + B + C = 180°$, this reduces to

$$\frac{-1}{4 \sin A \sin B \sin C} \begin{vmatrix} \sin^2 B + \sin^2 C - \sin^2 A, & \sin^2 A + \sin^2 C - \sin^2 B, & \sin^2 A + \sin^2 B - \sin^2 C \\ 1, & 1, & 1 \\ 2 \sin^2 A, & 2 \sin^2 B, & 2 \sin^2 C \end{vmatrix},$$

[1] See, for example, Cajori's *Theory of Equations*, pages 85–86.

[2] Stern, *Crelle's Journal*, volume 84, pages 216–218.

[3] *Proceedings Indiana Academy of Sciences*, 1914, page 440.

or

$$\frac{-1}{4\sin A\sin B\sin C}\begin{vmatrix} \sin^2 A+\sin^2 B+\sin^2 C, & \sin^2 A+\sin^2 B+\sin^2 C, & \sin^2 A+\sin^2 B+\sin^2 C \\ 1, & 1, & 1 \\ 2\sin^2 A, & 2\sin^2 B, & 2\sin^2 C \end{vmatrix}.$$

But this is equal to zero since two rows are alike after dividing out $\sin^2 A+\sin^2 B+\sin^2 C$. It is to be noticed also that the above determinant is equal to zero for any values whatever of A, B, C provided only that two of them are alike.

Also solved by ELIJAH SWIFT, G. W. HARTWELL, H. POLISH, R. M. MATHEWS, H. L. AGARD, H. S. UHLER, CLIFFORD N. MILLS, W. W. BURTON, CARL A. W. STROM, A. M. KENYON, J. H. WEAVER, and A. H. WILSON.

CALCULUS.

387. Proposed by C. N. SCHMALL, New York City.

Show that the volume bounded by the cone $x^2+y^2=(a-z)^2$ and the planes $x=0$, $x=z$ is $\frac{2}{3}a^3$.

I. Solution by A. M. HARDING, University of Arkansas.

The projection on the XY-plane of the curve of intersection of the cone and the plane $z=x$ is $y^2=a^2-2ax$.

If we change this equation to polar coördinates we obtain

$$\rho=\frac{a}{1+\cos\theta}=\frac{a}{2}\sec^2\frac{\theta}{2}.$$

Hence,

$$\frac{v}{2}=\int_0^{\pi/2}\int_0^{a/2\sec^2\theta/2}\int_z^{a-\sqrt{x^2+y^2}}dz\cdot\rho d\rho d\theta=\int_0^{\pi/2}\int_0^{a/2\sec^2\theta/2}(a-\sqrt{x^2+y^2}-x)\rho d\rho d\theta$$

$$=\int_0^{\pi/2}\int_0^{a/2\sec^2\theta/2}(a\rho-\rho^2-\rho^2\cos\theta)d\rho d\theta=\frac{a^3}{24}\int_0^{\pi/2}\sec^4\frac{\theta}{2}d\theta=\frac{a^3}{9}.$$

Hence the entire volume is $v=\frac{2}{3}a^3$.

II. Solution by Geo. W. HARTWELL, Hamline University.

If this volume is sliced parallel to the xy plane, the sections between $z=0$ and $z=a/2$ will be segments of circles and from $z=a/2$ to $z=a$ semicircles.

Integrating then we have

$$\int_0^{a/2}\left[\tfrac{1}{2}\pi(a-z)^2+z\sqrt{a^2-2az}-(a-z)^2\cos^{-1}\frac{z}{a-z}\right]dz+\int_{a/2}^{a}\tfrac{1}{2}\pi(a-z)^2dz$$

$$=\int_0^{a}\tfrac{1}{2}\pi(a-z)^2dz+\int_0^{a/2}\left[z\sqrt{a^2-2az}-(a-z)^2\cos^{-1}\frac{z}{a-z}\right]dz.$$

Integrating we have

$$\left[-\tfrac{1}{3}\pi(a-z)^3\right]_{z=0}^{z=a}+\left[-\frac{(2a^2+6az)(a^2-2az)^{1/2}}{30a^2}+\frac{(a-z)^3}{3}\cos^{-1}\frac{z}{a-z}-\frac{a^2\sqrt{a^2-2az}}{3}\right.$$

$$\left.+\frac{(2a^2+2az)\sqrt{a^2-2az}}{9}-\frac{(2a^2+2az+3z^2)\sqrt{a^2-2az}}{45}\right]_{z=0}^{z=(a/2)}=\tfrac{2}{3}a^3.$$

Also solved by H. L. AGARD, FRANK R. MORRIS, NORMAN ANNING, H. S. UHLER, and the PROPOSER.

388. Proposed by PAUL CAPRON, U. S. Naval Academy.

If $f(x, y)=0$ represents a curve having a simple tangency to the axis of x at the origin, the

value of $x^2/2y$, derived from $f(x, y) = 0$, and evaluated for $x = 0$, $y = 0$, will be the radius of curvature at the origin; or if the curve is similarly tangent to the y-axis at the origin, $y^2/2x$, evaluated for $x = 0$, $y = 0$, is the radius of curvature at the origin.

SOLUTION BY A. M. HARDING, University of Arkansas.

The equation of any curve having simple tangency to the axis of x at the origin may be written in the form

$$f(x, y) = Ay + \tfrac{1}{2}Bx^2 + Cxy + \tfrac{1}{2}Dy^2 + \cdots = 0,$$

where $A, B, C, D, \cdots$ are constants and $A \neq 0, B \neq 0$.

The radius of curvature at any point is given by

$$r^2 = \frac{[f_x^2 + f_y^2]^3}{[f_{xx}f_y^2 - 2f_{xy}f_xf_y + f_{yy}f_x^2]^2}, \qquad \text{where} \qquad f_{xy} = \frac{\partial^2 f}{\partial x \partial y}, \text{ etc.}$$

When $x = 0$ and $y = 0$, we find

$$f_x = 0, \quad f_y = A, \quad f_{xx} = B, \quad f_{xy} = C, \quad f_{yy} = D.$$

Substituting, we obtain $r^2 = A^2/B^2$ or $r = A/B$. Dividing the given equation by B, we have

$$\frac{A}{B} + \frac{x^2}{2y} + Cx + \frac{Dy}{2B} + \cdots = 0.$$

Letting x and y approach zero,

$$\frac{A}{B} = -\left[\frac{x^2}{2y}\right]_{\substack{x=0\\y=0}}.$$

Hence,

$$r = -\left[\frac{x^2}{2y}\right]_{\substack{x=0\\y=0}}.$$

In the second case the method is the same and the equation has the form

$$f(x, y) = Ax + \tfrac{1}{2}Bx^2 + Cxy + \tfrac{1}{2}Dy^2 + \cdots = 0.$$

Also solved by HORACE OLSON in a special form and with incorrect interpretation of "simple tangency."

389. Proposed by FRANK R. MORRIS, Glendale, Calif.

A man is at the southeast corner of a section of land and wishes to walk to the opposite corner in the least possible time. A circular track with a radius of $1/\pi$ miles is located in the section tangent to the west line at a point 120 rods from the south line. Conditions are such that he can walk at the rate of 4 miles an hour inside the track and 3 miles an hour outside the track. What course should he choose and how long is it?

I. SOLUTION BY H. S. UHLER, Yale University.

Since the problem involves rectilinear motion at different speeds it is a question of refraction and can be solved at once by the methods of geometrical optics; for, by Fermat's principle, the time taken by light in going from the southeast to the northwest corner of the section will be either a minimum or a maximum (in this case, a minimum). The index of refraction of the medium outside the circle relative to the medium inside the circle is 4/3 (water and air, say). Hence, the "optical invariant" is

$$3 \sin i = 4 \sin r. \tag{1}$$

By hypothesis, (Fig. 1).

$\overline{OW} = \overline{WN} = 1$ mile. $\overline{CA} = \overline{CB} = \overline{CT} = 1/\pi$ miles, and $\overline{PC} = 120$ rods $= \frac{3}{4}$ mile.

The $\triangle ACB$ is isosceles so that $\angle CBA = \angle CAB \equiv i$. Hence, by equation (1) $\angle EBN = \angle DAO \equiv r$.

The projection of the broken line $OPCA$ on a diameter $\overline{FG}$ perpendicular to $\overline{OA}$ equals zero.

Hence,

$$\overline{IH} + \overline{HC} + \overline{CI} = 0; \qquad \overline{IH} = \overline{JP} = \overline{OP} \sin\phi = (1 - 1/\pi)\sin\phi;$$

$$\overline{HC} = -\overline{PC}\cos\phi = -\tfrac{3}{8}\cos\phi; \qquad \overline{CI} = -\overline{AC}\sin r = -1/\pi \sin r.$$

Therefore,

$$8(\pi - 1)\sin\phi - 3\pi\cos\phi - 8\sin r = 0. \tag{2}$$

In like manner, the projection of the broken line $NTCB$ on a diameter $\overline{KL}$ perpendicular to $\overline{NB}$ leads to

$$8\sin(\phi + 2i - 2r) - 5\pi\cos(\phi + 2i - 2r) + 8\sin r = 0. \tag{3}$$

Equations (1), (2), and (3) determine the acute angles ϕ, i, and r uniquely. Elimination of i and r would lead to a rational equation of degree 16 in $\tan\phi$. The coefficients would involve inconveniently high powers of π and the required root would have to be found by some method of *approximation*, such as Horner's. Consequently no mathematical self-respect is lost and an enormous amount of time is saved by employing the following process of approximation.

Assume a numerical value of ϕ and calculate r from equation (2). Next evaluate i from (1). Then substitute ϕ, i, and r in (3). Repeat until the left member of (3) vanishes. It will then be found that

$$\phi = 37^\circ\ 33'\ 10''; \qquad i = 29^\circ\ 40'\ 27''; \qquad r = 21^\circ\ 47'\ 45''.$$

Let the rectangular coördinates of A and B be (x_1, y_1) and (x_2, y_2) respectively, where abscissas and ordinates are reckoned positive westward and northward in the order named.

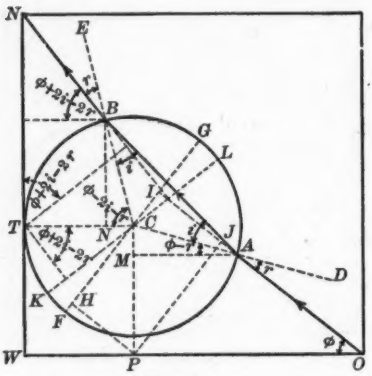


FIG. 1.

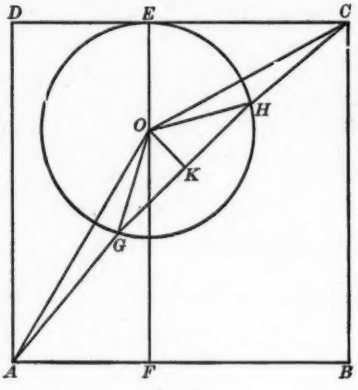


FIG. 2.

$$x_1 = \overline{OP} - \overline{AM} = \left(1 - \frac{1}{\pi}\right) - \frac{1}{\pi}\cos(\phi - r) = 0.375342 \text{ mile},$$

$$y_1 = x_1 \tan\phi = 0.288561 \text{ mile}.$$

$$l_1 = \overline{OA} = x_1 \sec\phi = 0.473444 \text{ mile}, \qquad l_2 = \overline{AB} = \frac{2}{\pi}\cos i = 0.553130 \text{ mile},$$

$$x_2 = \overline{OW} - (\overline{CT} - \overline{CN}) = 1 - \frac{1}{\pi} + \frac{1}{\pi}\cos(\phi + 2i - r) = 0.763511 \text{ mile}.$$

$$y_2 = \overline{PC} + \overline{NB} = \frac{3}{8} + \frac{1}{\pi}\sin(\phi + 2i - r) = 0.682615 \text{ mile},$$

$$l_3 = \overline{BN} = \overline{NT}\sec(\phi + 2i - 2r) = 0.395803 \text{ mile}.$$

Hence,

$$l_1 + l_2 + l_3 = 1 \text{ mi. } 135 \text{ rd. } 0 \text{ yd. } 2 \text{ ft. } 8 \text{ in.},$$

$$t_1 = 9 \text{ min. } 28.13 \text{ sec.}, \qquad t_2 = 8 \text{ min. } 17.82 \text{ sec.}, \qquad t_3 = 7 \text{ min. } 54.96 \text{ sec.}$$

Hence,

$$t_1 + t_2 + t_3 = 25 \text{ min. } 40.9 \text{ sec.}$$

Finally, the numerical data satisfy the following check equations, obtained by projection of the path on the coördinate axes,

$$l_1 \cos\phi + l_2 \cos(\phi + i - r) + l_3 \cos(\phi + 2i - 2r) = 1,$$

$$l_1 \sin\phi + l_2 \sin(\phi + i - r) + l_3 \sin(\phi + 2i - 2r) = 1.$$

II. Solution by A. H. Holmes, Brunswick, Maine.

Let $ABCD$ (Fig. 2) be the section of land, 1 mile square, A being the southeast corner, B the northeast corner, C the northwest corner, and D the southwest corner. On DC, take $DE = 120$ rods or $\frac{3}{4}$ of a mile and draw EF perpendicular to AB at F. On EF, take $EO = 1/\pi$ of a mile to point O.

With O as a center and EO as radius describe a circle. Draw AO and CO. Let AG be the course to the circumference of track, GH the course inside the track, and HC the remainder of the course to the northwest corner C.

Draw OK perpendicular to GH at K. The ratio of GH to $AG + CH$ is the greatest when OK bisects the angle AOC. Therefore, for a minimum, the angles AOG and COH are equal.

Put $AG = x$, $CH = y$, $GH = z$, $AO = a$, $CO = b$, $1/\pi = r$, $AOG = COH = \theta$, and $AOC = 2m$.

Then

$$\frac{x + y}{3} + \frac{z}{4} = \text{minimum.}$$

In $\triangle AGO$ and CHO

$$\cos\theta = \frac{a^2 + r^2 - x^2}{2ar}, \qquad \cos\theta = \frac{b^2 + r^2 - y^2}{2br}.$$

Hence $x = \sqrt{a^2 + r^2 - 2ar\cos\theta}$, $y = \sqrt{b^2 + r^2 - 2br\cos\theta}$, and $z = 2r\sin(m - \theta)$ in which $a = .778027$, $b = .701390$, $r = .318310$, and $m = 72° 54' 9''$.

Multiplying x and y by 4 and z by 3, reducing and differentiating,

$$\frac{2a\sin\theta}{\sqrt{a^2 + r^2 - 2ar\cos\theta}} + \frac{2b\sin\theta}{\sqrt{b^2 + r^2 - 2br\cos\theta}} = 3\cos(m - \theta).$$

Whence, $\theta = 12° 33' 9''$.

Knowing θ, the required course in distances and directions is easily found.

From A to G, N. 52° 45′ 48″ W. 151.17 rods; G to H, N. 44° 5′ 13″ W. 177.03 rods; H to C, N. 37° 1′ 51″ W. 126.96 rods, the length of the whole course being 455.16 rods, and made in the minimum time of 25 minutes 40.8 seconds.

MECHANICS.

303. Proposed by Clifford N. Mills, Brookings, South Dakota.

A pile driver weighing 500 pounds falls through 10 feet and drives a pile weighing 400 pounds 3 inches into the ground. Show that the average force of the blow is 11,111⅑ pounds.

Solution by W. H. Williams, Oakland, California.

Let m = mass of driver in pounds, m' = mass of pile in pounds, v = velocity of driver at instant of impact in feet per second, h = height of fall of driver in feet, d = distance pile is driven into ground in feet, r = average retardation of pile and driver after impact in feet per second per second, t = time from impact until rest in seconds, and g = acceleration due to gravity in feet per second per second, F = average force of blow.

Then mv = momentum of driver at impact = momentum of driver + pile immediately after impact; $mv/(m + m')$ = common velocity of driver and pile immediately after impact.

Hence,

$$\frac{mv}{(m + m')r} = t, \qquad \text{and} \qquad d = \tfrac{1}{2}rt^2 = \tfrac{1}{2}r\,\frac{m^2v^2}{(m + m')^2r^2} = \tfrac{1}{2}\,\frac{m^2v^2}{(m + m')^2r},$$

or

$$r = \tfrac{1}{2}\frac{m^2v^2}{(m+m')^2d}.$$

But $v^2 = 2gh$, and hence,

$$r = \frac{m^2gh}{(m+m')^2d}.$$

Finally,

$$F = (m+m')r = \frac{m^2gh}{(m+m')d} \text{ poundals} = \frac{m^2h}{(m+m')d} \text{ pounds.}$$

Substituting values,

$$F = \frac{500 \times 500 \times 10}{(500+400) \times \frac{1}{4}} = 11{,}111\tfrac{1}{9} \text{ lbs.}$$

Also solved by A. M. HARDING and J. A. CAPARO. Erroneously solved by HERBERT N. CARLETON.

305. Proposed by B. J. BROWN, Victor, Colorado.

A particle is to be projected so as to graze the top of a wall h feet high, at a distance of a feet from the point of projection, and to strike the ground at a distance b feet from the foot of the wall. Find the velocity of projection, and the inclination of the path to the horizontal, at the ground and at the top of the wall. I. C. S. 1903.

SOLUTION BY EMMA M. GIBSON, Assistant at Drury College.

The equation of the path of the particle is

$$y = x\tan\alpha - \tfrac{1}{2}g\frac{x^2}{v_0^2\cos^2\alpha},$$

where v_0 is the initial velocity and α the angle of projection.

Knowing the points (a, h), $(a+b, 0)$ to be on the curve the following satisfied relations exist:

$$h = a\tan\alpha - \tfrac{1}{2}g\frac{a^2}{v_0^2\cos^2\alpha} \tag{1}$$

and

$$(a+b)\tan\alpha - \tfrac{1}{2}g\frac{(a+b)^2}{v_0^2\cos^2\alpha} = 0. \tag{2}$$

From equation (2),

$$v_0^2 = \frac{(a+b)g}{2\sin\alpha\cos\alpha}, \quad \text{or} \quad v_0 = \sqrt{\frac{(a+b)g}{\sin 2\alpha}}.$$

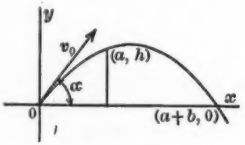
Substituting this value of v_0 in (1) and solving for $\tan\alpha$,

$$\tan\alpha = \frac{(a+b)h}{ab}.$$

Hence,

$$\alpha = \tan^{-1}\left[\frac{(a+b)h}{ab}\right],$$

the inclination of the path at the ground.



The equation of the tangent to the curve at any point (x_1, y_1) is

$$\frac{y+y_1}{2} = \frac{x+x_1}{2}\tan\alpha - \tfrac{1}{2}g\frac{x_1x}{v_0^2\cos^2\alpha}.$$

At (a, h) the tangent becomes

$$y + h = (x+a)\tan\alpha - \frac{agx}{v_0^2\cos\alpha} = \left[\frac{v_0^2\sin\alpha\cos\alpha - ag}{v_0^2\cos^2\alpha}\right]x + a\tan\alpha.$$

The inclination of this line,

$$\tan^{-1}\left[\frac{v_0^2\sin\alpha\cos\alpha - ag}{v^2\cos^2\alpha}\right] = \tan^{-1}\left[\left(\frac{b-a}{ab}\right)h\right],$$

is the inclination of the path to the horizontal at the top of the wall.

Also solved by H. C. FEEMSTER, J. A. CAPARO, CLIFFORD N. MILLS, and HORACE OLSON. Erroneously solved by HERBERT N. CARLETON.

306. Proposed by EMMA M. GIBSON, Drury College.

A sphere is composed of a solid homogeneous hemisphere and a very thin hemispherical shell of equal mass. What is the greatest inclination of a rough plane on which the sphere can just rest in equilibrium?

SOLUTION BY PAUL CAPRON, U. S. Naval Academy.

Let the radius of the sphere be a, the center at O, the center of gravity at G; then

$$OG = \frac{1}{2}\left(\frac{a}{2} - \frac{3a}{8}\right) = \frac{a}{16};$$

however the sphere is placed, G is on a circumference of radius $a/16$, centered at O. If the sphere is in equilibrium, G is vertically over the contact, C, of the sphere with the inclined plane on which it rests; the inclination of the plane is the angle COG, which has its greatest value when CG is tangent to the little circle; this value is $\sin^{-1} 1/16$. Hence, the greatest inclination of the plane is $\sin^{-1} 1/16$ or $\tan^{-1} \mu$ (μ being the coefficient of friction) whichever happens to be the less.

Also solved by J. A. CAPARO. Erroneously solved by HERBERT N. CARLETON.

NUMBER THEORY.

226. Proposed by ELBERT H. CLARKE, Purdue University.

If 0! is taken equal to 1, and if k is any positive integer greater than or equal to 2, show that

$$\sum_{n=0}^{\infty} \frac{n!}{(k+n)!} = \frac{1}{k-1} \cdot \frac{1}{(k-1)!}.$$

I. SOLUTION BY S. A. JOFFE, New York City.

Since

$$\frac{n!}{(k+n)!} = \frac{1 \cdot 2 \cdot 3 \cdots n}{1 \cdot 2 \cdot 3 \cdots n(n+1)(n+2) \cdots (n+k)} = \frac{1}{(n+1)(n+2) \cdots (n+k)},$$

we see that

$$\sum_{n=0}^{\infty} \frac{n!}{(k+n)!} = \sum_{n=0}^{\infty} \frac{1}{(n+1)(n+2) \cdots (n+k)}. \tag{1}$$

Now, the series in the second member, each term of which is the reciprocal of a factorial expression, may be easily summed by an elementary theorem in finite differences or by the following equivalent simple method.

Noticing that

$$\frac{1}{(n+1)(n+2) \cdots (n+k-1)} - \frac{1}{(n+2)(n+3) \cdots (n+k)}$$
$$= \frac{(n+k)-(n+1)}{(n+1)(n+2) \cdots (n+k)} = \frac{k-1}{(n+1)(n+2) \cdots (n+k)},$$

we find that

$$\frac{1}{(n+1)(n+2) \cdots (n+k)}$$
$$= \frac{1}{k-1}\left[\frac{1}{(n+1)(n+2) \cdots (n+k-1)} - \frac{1}{(n+2)(n+3) \cdots (n+k)}\right].$$

Making in the last equation n equal successively to 0, 1, 2, $\cdots$, we get

$$\frac{1}{1\cdot 2\cdots k} = \frac{1}{k-1}\left[\frac{1}{1\cdot 2\cdots(k-1)} - \frac{1}{2\cdot 3\cdots k}\right],$$

$$\frac{1}{2\cdot 3\cdots(k+1)} = \frac{1}{k-1}\left[\frac{1}{2\cdot 3\cdots k} - \frac{1}{3\cdot 4\cdots(k+1)}\right],$$

$$\frac{1}{3\cdot 4\cdots(k+2)} = \frac{1}{k-1}\left[\frac{1}{3\cdot 4\cdots(k+1)} - \frac{1}{4\cdot 5\cdots(k+2)}\right],$$

$$\cdots\cdots\cdots

Now assume the given statement true if $k = r$; *i. e.*, that

$$\frac{1}{r-1}\cdot\frac{1}{(r-1)!}=\frac{0!}{r!}+\frac{1!}{(r+1)!}+\frac{2!}{(r+2)!}+\cdots+\frac{n!}{(r+n)!}+\cdots. \tag{3}$$

Transposing the first term of the second member of (3),

$$\frac{1}{r-1}\cdot\frac{1}{(r-1)!}-\frac{0!}{r!}=\frac{1!}{(r+1)!}+\frac{2!}{(r+2)!}+\cdots+\frac{r!}{(r+n)!}+\cdots, \tag{4}$$

where the second member is an absolutely convergent series. But

$$\frac{1}{r-1}\cdot\frac{1}{(r-1)!}-\frac{0!}{r!}=\frac{\frac{r}{r-1}-1}{r!}=\frac{1}{r!(r-1)}.$$

Accordingly, (4) becomes

$$\frac{1}{r!(r-1)}=\frac{1!}{(r+1)!}+\frac{2!}{(r+2)!}+\frac{3!}{(r+3)!}+\cdots+\frac{n!}{(r+n)!}+\frac{(n+1)!}{(r+n+1)!}+\cdots. \tag{5}$$

Subtracting (5) from (3) gives, since the series are convergent,[1]

$$\begin{aligned}\frac{1}{r-1}\cdot\frac{1}{(r-1)!}-\frac{1}{r!(r-1)}&=\left(\frac{0!}{r!}-\frac{1!}{(r+1)!}\right)+\left(\frac{1!}{(r+1)!}-\frac{2!}{(r+2)!}\right)\\&\quad+\left(\frac{2!}{(r+2)!}-\frac{3!}{(r+3)!}\right)+\cdots+\left(\frac{n!}{(r+n)!}-\frac{(n+1)!}{(r+n+1)!}\right)+\cdots\\&=\frac{1!r}{(r+1)!}+\frac{1!(2+r)-2!}{(r+2)!}+\frac{2!(3+r)-3!}{(r+3)!}+\cdots+\frac{n!(n+1+r)-(n+1)!}{(r+n+1)!}+\cdots\\&=r\left\{\frac{0!}{(r+1)!}+\frac{1!}{(r+2)!}+\frac{2!}{(r+3)!}+\cdots+\frac{n!}{(r+n+1)!}+\cdots\right\}.\end{aligned} \tag{6}$$

Now the first member of (6) may be simplified:

$$\frac{1}{r-1}\cdot\frac{1}{(r-1)!}-\frac{1}{r!(r-1)}=\frac{r-1}{r!(r-1)}=\frac{1}{r!}.$$

Accordingly, dividing each member of (6) by r,

$$\frac{1}{r!r}=\frac{0!}{(r+1)!}+\frac{1!}{(r+2)!}+\frac{2!}{(r+3)!}+\cdots+\frac{n!}{(r+n+1)!}+\cdots.$$

But this is exactly what (1) becomes if $r+1$ is substituted for k. Hence, if (1) is true for a particular integral value of k ($k > 1$), then it is true for k equal to the next greater integer. But (1) was shown true if $k = 2$, accordingly, it is true for $k = 3$; if for $k = 3$, then for $k = 4$; etc.

Also solved by HORACE OLSON, FRANK IRWIN, and ELIJAH SWIFT.

[1] Pierpont, *loc. cit.*, article 111.

QUESTIONS AND DISCUSSIONS.

EDITED BY U. G. MITCHELL, University of Kansas.

DISCUSSIONS.

I. RELATING TO THE INTERPRETATION OF THE DEGENERATE CONICS.

BY H. W. BRINKMANN, Student in the Polytechnic High School, Riverside, Calif.

In textbooks on analytic geometry the degenerate cases which appear on analyzing the general equation of the second degree are usually treated from the algebraic side alone. It is easily shown, however, that all the degenerate cases can be made to fit precisely into the definitions of the proper conics.

Consider first the case of two intersecting lines. These can be interpreted as a hyperbola as follows. If we take the bisector of one angle between them, the ratio of the distance from any point on either line to their point of intersection and the distance to the bisector is constant, viz., the secant of one half the angle between the lines. In other words, the lines can be called an hyperbola whose focus is the intersection, whose directrix is the bisector of the angle between the lines, and whose eccentricity is the secant of one half that angle. Since the secant of an angle can never be less than unity, it is clear that the lines can represent only an hyperbola.

Evidently the bisector of the second angle could be taken as directrix with the same focus. Thus we see that the pair of lines are really two coincident, conjugate hyperbolas.

The second degenerate case, namely a point, is most easily interpreted as an ellipse whose semi-axes are zero. For, if the point is also regarded as the coincident foci of the ellipse, it is clear that the sum of the distances from the point to the foci is constant, namely zero. Therefore the point is an ellipse whose semi-axes are zero.

The only degenerate case that remains is that of two parallel lines. If these are coincident, it evidently represents a parabola whose directrix is any line perpendicular to it and whose focus is their point of intersection, for then the distance from any point on this line to the directrix equals the distance from that point to the focus.

If the lines are non-coincident they are best interpreted as the limiting case of a pair of intersecting lines, if the point of intersection moves indefinitely to one side in the plane. The angle between those lines is then zero and its secant is unity. Hence the hyperbola (intersecting lines) has degenerated into a parabola, for its eccentricity is unity.

It may also be noticed that for the equation of a conic where the focus lies on the directrix we get

$$x^2(1 - e^2) + y^2 = 0$$

where e is the eccentricity. For, taking the directrix as the y-axis and the focus

as the origin, we have $\sqrt{x^2+y^2}$ as the distance from any point (x, y) to the focus and x as the distance from that point to the directrix. Hence, if the point is on the locus,

$$\sqrt{x^2+y^2} = ex$$

and squaring we get the required equation.

For $e > 1$, $e = 1$, $e < 1$, this reduces to the equations of two intersecting lines, two coincident lines and a point respectively.

II. TWO USEFUL RELATIONS IN TRIGONOMETRY.

By Albert Babbitt, University of Minnesota.

If ABC be any triangle, and if θ be the angle which a straight line CP (Fig. 1) drawn from the vertex C makes with the base of the triangle, and if we let $\angle PCA = \alpha$, $\angle BCP = \beta$, $AP = m$, $PB = n$, we then have

$$(m+n)\cot\theta = m\cot\alpha - n\cot\beta.$$

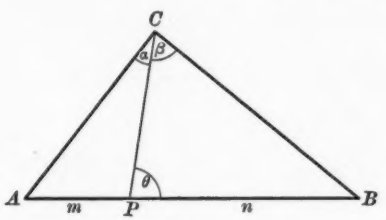


Fig. 1.

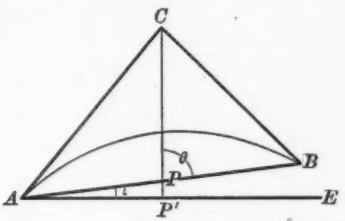


Fig. 2.

The proof of this trigonometric relation is very simple.[1] From $\triangle ACP$, we have

$$\frac{CP}{m} = \frac{\sin A}{\sin\alpha},$$

hence,

$$CP = \frac{m\sin A}{\sin\alpha} = \frac{m\sin(\theta-\alpha)}{\sin\alpha} = m(\sin\theta\cot\alpha - \cos\theta). \qquad (1)$$

Similarly, from $\triangle PCB$, we have

$$CP = \frac{n\sin B}{\sin\beta} = \frac{n\sin(\theta+\beta)}{\sin\beta} = n(\sin\theta\cot\beta + \cos\theta). \qquad (2)$$

From (1) and (2) it follows that

$$m(\sin\theta\cot\alpha - \cos\theta) = n(\sin\theta\cot\beta + \cos\theta).$$

Simplifying and dividing through by $\sin\theta$ (since $\sin\theta \neq 0$), we get

$$(m+n)\cot\theta = m\cot\alpha - n\cot\beta.$$

[1] Other proofs are given in, for instance, Loney's *Elements of Statics and Dynamics*, tenth edition (1896), page 92. Editor.

Using the same figure and notation, we can derive in a similar way the relation,

$$(m + n) \cot \theta = n \cot A - m \cot B.$$

These trigonometric relations facilitate the solution of some problems in physics.

For example, having given the direction and velocity of projection, let it be required to find the velocity with which a projectile would strike an oblique plane.

If we denote by i the inclination of the plane to the horizon (Fig. 2), and apply the second relation above derived, noting that tangents at two points A and B of a parabola meet on the diameter bisecting AB, we get

$$\cot A - \cot B = 2 \tan i.$$

Hence, we determine the angle B, since the angles A and i are known. But, if AC and BC are tangents at two points A and B on a parabolic trajectory, and V_1 and V_2 the corresponding velocities, then

$$\frac{V_1}{V_2} = \frac{\sin B}{\sin A} \quad \text{and} \quad V_2 = \frac{V_1 \sin A}{\sin B},$$

and the required velocity is determined.

THE CHARTER MEMBERSHIP OF THE ASSOCIATION.

The By-Laws of the Association provided that those who should be elected to membership before April 1, 1916, should constitute the list of Charter Members. Owing to circumstances beyond our control the issue of the Monthly containing the report of the Columbus meeting and the announcement of the conditions of membership could not be distributed to the public till well into February. Similar delays of subsequent issues likewise prevented the prompt distribution of other information, so that the great flood of applications came late in March, thus making it impossible to complete the report until the present time.

The following is a preliminary report of the charter membership giving only the name of each member and the institution represented. As soon as possible there will be published a complete directory, giving the official position and mailing address of each member. Meanwhile, we invite careful scrutiny of the present list in order to detect any omissions or errors which may have escaped the attention of the secretary in the arduous task of checking up the returns.

It was assumed that all who joined in the call for the Columbus meeting, or took part in that meeting, would consider themselves eligible to charter membership, and the executive committee of the Council so ruled. From eighteen of these five hundred persons we had not, on going to press, received any expression of their intention, but we are provisionally including their names in the list below, pending the receipt of such information.

ALABAMA.

F. L. Carmichael, University of Alabama.
F. E. Chapman, Southern University.
B. H. Crenshaw, Alabama Polytechnic Institute.
T. R. Eagles, Howard College.
W. V. Luckie, Sulligent Public School.
J. F. Messick, Alabama Polytechnic Institute.
H. A. Sayre, University of Alabama.
B. L. Shi, Alabama Polytechnic Institute.

ARIZONA.

G. M. Butler, University of Arizona.
G. H. Cresse, University of Arizona.
C. O. Lampland, Lowell Observatory.
H. B. Leonard, University of Arizona.

ARKANSAS.

Flora Armitage, Little Rock High School.
J. A. Bigbee, Little Rock High School.
P. N. Bragg, Helena High School.
G. W. Droke, University of Arkansas.
A. M. Harding, University of Arkansas.
H. L. McAlister, Ouachita College.
W. L. Miser, University of Arkansas.
W. M. Steirnagle, Manilla.

CALIFORNIA.

Gertrude E. Allen, San Diego Junior College.
Paul Arnold, University of Southern California.
F. A. Ballaseyus, Stockton High School.
B. A. Bernstein, University of California.
H. F. Blichfeldt, Stanford University.
F. P. Brackett, Pomona College.
Myrtie Collier, Los Angeles Normal School.
S. R. Cook, College of the Pacific.
F. E. Crofts, Lowell High School, San Francisco.
F. J. Dick, Raja Yoga College.
H. O. Eggen, Santa Ana Junior College.
Sadie L. Gilmore, Antioch High School.
R. L. Green, Stanford University.
M. W. Haskell, University of California.
J. E. Higdon, Life Insurance, Los Angeles.
L. M. Hoskins, Stanford University.
Frank Irwin, University of California.
D. N. Lehmer, University of California.
C. T. Levy, University of California.
R. M. Mathews, Polytechnic High School, Riverside.
A. L. McCarty, Lowell High School, San Francisco.
G. F. McEwen, Scripps Biological Institution.
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F. R. Morris, Glendale Union High School.
C. A. Noble, University of California.
E. W. Ponzer, Stanford University.
F. D. Posey, Pipe Line Company, Lebec.
T. M. Putnam, University of California.
W. P. Russell, Pomona College.
T. J. J. See, U. S. Naval Observatory.
H. W. Stager, Fresno Junior College.
C. M. Titus, University Farm School.
Jean Tuttle, Watsonville High School.
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CANADA.

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N. A. Draxten, Langenburg, Sask.
William Findlay, McMaster University.
C. F. Gummer, Queen's University.
H. R. Kingston, University of Manitoba.
G. H. Ling, University of Saskatchewan.
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D. A. Murray, McGill University.
I. R. Pounder, University of Toronto.
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E. W. Sheldon, University of Alberta.
L. A. H. Warren, University of Manitoba.

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E. L. Brown, North Side High School, Denver.
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C. A. Short, Delaware College.

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Director, Department of Terrestrial Magnetism, Washington.
Harry English, Washington High Schools.
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M. Cecilia Mangold, Trinity College.
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P. E. Hemke, Georgia School of Technology.
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Amy F. Preston, Agnes Scott College.
Douglas Rumble, Emory College.
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D. M. Smith, Georgia School of Technology.
D. L. Stamy, Georgia School of Technology.
R. P. Stephens, University of Georgia.
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Chester Snow, University of Idaho.

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R. E. Wilson, Northwestern University.
Alice Winbigler, Monmouth College.
Olice Winter, Harrison Technical High School, Chicago.
C. H. Yeaton, Northwestern University.
J. W. A. Young, University of Chicago.

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F. H. Hodge, Franklin College.
E. N. Johnson, Butler College.
A. M. Kenyon, Purdue University.
Alexander Knisely, Columbia City.
R. S. Lawrence, Hanover College.
Charles Leckrone, Manchester College.
D. A. Lehman, Goshen College.
W. V. Lovitt, Purdue University.
W. O. Mendenhall, Earlham College.
E. T. Motts, South Bend.
C. K. Robbins, Purdue University.
D. A. Rothrock, Indiana University.
George Spitzer, Purdue University.
O. D. Tyner, High School, Ft. Wayne.
K. P. Williams, Indiana University.
W. A. Zehring, Purdue University.

IOWA.

Edna Allen, Iowa State Teachers College.
R. P. Baker, State University of Iowa.
W. E. Beck, State University of Iowa.
G. A. Chaney, State College of Agriculture.
Julia T. Colpitts, State College of Agriculture.
I. S. Condit, Iowa State Teachers College.
S. A. Corey, Wapello Coal Company, Albia.
Marian E. Daniells, State College of Agriculture.
C. W. Emmons, Simpson College.
Fay Farnum, State College of Agriculture.
Cornelius Gouwens, State University of Iowa.
S. M. Hadley, Penn College.
Gertrude A. Herr, State College of Agriculture.
Daniel Kreth, Wellman.
R. B. McClenon, Grinnell College.
F. M. McGaw, Cornell College.
Nina Madson, State College of Agriculture.
I. F. Neff, Drake University.
Mrs. Mary B. Norton, Cornell College.
E. A. Pattengill, State College of Agriculture.
J. F. Reilly, State University of Iowa.
Maria M. Roberts, State College of Agriculture.
J. E. Robertson, Highland Park College.
W. J. Rusk, Grinnell College.
B. F. Simonson, Upper Iowa University.
A. G. Smith, State University of Iowa.
S. G. Stein, Muscatine.
E. H. Thomas, Tabor College.
E. L. Thompson, Burlington High School.
R. N. Van Horne, Sioux City.
C. W. Wester, State University of Iowa.
John Zimmerman, Dubuque German College and Seminary.

KANSAS.

W. H. Andrews, State Agricultural College.
C. H. Ashton, University of Kansas.
Talmon Bell, Cooper College.
T. L. Bouse, Campbell College.
M. R. Bowerman, State Agricultural College.
V. B. Caris, State Manual Training Normal.
Lucy T. Dougherty, Kansas City High School.
Ottilia W. Dueker, Friends University.
Elizabeth G. Flagg, Kansas City High School.
W. H. Garrett, Baker University.
P. W. Harnly, University of Kansas.
W. A. Harshbarger, Washburn College.
A. J. Hoare, Fairmount College.
K. J. Holzinger, University of Kansas.
Emma Hyde, Kansas City High School.
Jessie M. Jacobs, University of Kansas.
H. E. Jordan, University of Kansas.
A. W. Larsen, University of Kansas.
Solomon Lefschetz, University of Kansas.
Theodore Lindquist, State Normal School.
T. E. Mergendahl, College of Emporia.
U. G. Mitchell, University of Kansas.
C. A. Nelson, University of Kansas.
Mary W. Newson, Washburn College.
H. N. Olson, Bethany College.
H. E. Porter, State Agricultural College.
B. L. Remick, State Agricultural College.
D. H. Richert, Bethel College.

J. A. G. Shirk, State Manual Training School.
E. M. Stahl, Midland College.
L. L. Steimley, University of Kansas.
E. B. Stouffer, University of Kansas.
W. T. Stratton, State Agricultural College.
J. N. Van der Vries, University of Kansas.
J. J. Wheeler, University of Kansas.
A. E. White, State Agricultural College.
Ella Woodyard, Kansas City High School.

KENTUCKY.

P. P. Boyd, University of Kentucky.
J. M. Davis, University of Kentucky.
J. E. Dotterer, University of Kentucky.
Henry Lloyd, Transylvania College.
H. R. Phalen, Berea College.
E. L. Rees, University of Kentucky.
A. L. Rhoton, Georgetown College.

LOUISIANA.

A. Bogard, Shreveport High School.
J. T. Cater, Straight College.
A. B. Dinwiddie, Tulane University.
P. T. Hedges, Louisiana State Normal.
J. W. Nicholson, University of Louisiana.
Mary C. Spencer, Newcomb College.

MAINE.

A. S. Adams, Edward Little High School, Auburn.
T. B. Ashcraft, Colby College.
B. E. Carter, Colby College.
J. A. Colson, National Bank, Searsport.
M. T. Goodrich, High School, Bingham.
J. H. Hart, University of Maine.
A. H. Holmes, Brunswick.
Alice M. Lord, Portland High School.
W. E. Milne, Bowdoin College.
W. A. Moody, Bowdoin College.
G. E. Ramsdell, Bates College.
L. J. Reed, University of Maine.
Fannie H. Robinson, Bangor High School.
H. E. Trefethen, Colby College.

MARYLAND.

Clara L. Bacon, Goucher College.
Harry Bateman, Johns Hopkins University.
Lillian O. Brown, Hood College.
Paul Capron, U. S. Naval Academy.
A. B. Coble, Johns Hopkins University.
Abraham Cohen, Johns Hopkins University.
H. A. Converse, Baltimore Polytechnic Institute.
W. C. Eells, U. S. Naval Academy.
J. B. Eppes, U. S. Naval Academy.
J. N. Galloway, Baltimore Polytechnic Institute.
Angelo Hall, U. S. Naval Academy.
S. C. Harry, Friends School, Baltimore.
A. W. Hobbs, Baltimore.
L. S. Hulburt, Johns Hopkins University.
W. W. Johnson, U. S. Naval Academy.
Florence P. Lewis, Goucher College.
Frank Morley, Johns Hopkins University.
E. B. Morrow, Gilman Country School, Baltimore.
J. R. Musselman, Johns Hopkins University.
S. F. Norris, Baltimore City College.
C. H. Rawlins, Jr., Baltimore.
H. M. Robert, Jr., Baltimore.
R. E. Root, U. S. Naval Academy.
W. F. Shenton, Johns Hopkins University.
E. R. Smith, The Park School, Baltimore.
C. P. Sousley, Mount St. Mary's College.

MASSACHUSETTS.

H. L. Agard, Williams College.
F. H. Bailey, Massachusetts Institute of Technology.
Ida Barney, Smith College.
Ralph Beatley, Milton Academy.
Susan R. Benedict, Smith College.
G. D. Birkhoff, Harvard University.
C. L. Bouton, Harvard University.
H. C. Bradley, Massachusetts Institute of Technology.
L. A. Brigham, Boston University.
R. E. Bruce, Boston University.
E. S. Bryant, Everett High School.
J. A. Bullard, Worcester Polytechnic Institute.
A. D. Butterfield, Worcester Polytechnic Institute.
Eva Chandler, Wellesley College.
J. E. Clark, Springfield.
L. L. Conant, Worcester Polytechnic Institute.
M. Imogene Cook, Walnut Hill School, Natick.
J. L. Coolidge, Harvard University.
Lennie P. Copeland, Wellesley College.
H. N. Davis, Harvard University.
C. R. Duncan, Massachusetts Agricultural College.
T. C. Esty, Amherst College.
W. C. Esty, Amherst College.
G. W. Evans, Charlestown High School, Boston.
F. C. Ferry, Williams College.
H. D. Gaylord, Browne and Nichols School, Cambridge.
G. M. Green, Harvard University.
C. E. Haigler, Wentworth Institute, Boston.
J. G. Hardy, Williams College.
Olive C. Hazlett, Cambridge.
C. A. Hobbs, Watertown.
E. V. Huntington, Harvard University.
Dunham Jackson, Harvard University.
Ervin Kenison, Massachusetts Institute of Technology.
A. E. Kennelly, Harvard University.
Carl King, Wentworth Institute, Boston.
Edward Kircher, Harvard University.
B. B. Libby, Massachusetts Institute of Technology.
Joseph Lipka, Massachusetts Institute of Technology.
U. J. Lupien, Lowell Textile School.
Emilie N. Martin, Mount Holyoke College.
Helen A. Merrill, Wellesley College.
C. L. E. Moore, Massachusetts Institute of Technology.
M. M. S. Moriarty, Holyoke High School.
R. K. Morley, Worcester Polytechnic Institute.
Florence L. Munroe, Northampton High School.
C. E. Norwood, Dartmouth College.
G. D. Olds, Amherst College.

W. F. Osgood, Harvard University.
Anna J. Pell, Mount Holyoke College.
H. B. Phillips, Massachusetts Institute of Technology.
Gertrude E. Preston, Dana Hall School, Wellesley.
W. R. Ransom, Tufts College.
C. N. Reynolds, Jr., Cambridge.
George Rutledge, Massachusetts Institute of Technology.
Clara E. Smith, Wellesley College.
Sarah E. Smith, Mount Holyoke College.
J. J. Sullivan, Jr., Roxbury.
H. W. Tyler, Massachusetts Institute of Technology.
Roxana H. Vivian, Wellesley College.
G. L. Wagar, Mount Hermon Boys' School.
A. C. Washburne, Berkshire Life Insurance Company, Pittsfield.
George Wentworth, Brookline.
A. H. Wheeler, High School of Commerce, Worcester.
F. B. Williams, Clark University.
E. B. Wilson, Massachusetts Institute of Technology.
F. S. Woods, Massachusetts Institute of Technology.
Euphemia R. Worthington, Wellesley College.
W. C. Wright, Actuary, Medford.
Mabel M. Young, Wellesley College.

MICHIGAN.

J. W. Baldwin, University of Michigan.
W. W. Beman, University of Michigan.
J. W. Bradshaw, University of Michigan.
W. H. Butts, University of Michigan.
L. C. Emmons, Michigan Agricultural College.
A. G. Erickson, Michigan State Normal College.
E. B. Escott, Peninsular Life Insurance Company, Detroit.
J. P. Everett, Western State Normal School.
Peter Field, University of Michigan.
W. B. Ford, University of Michigan.
C. H. Forsyth, University of Michigan.
W. Van N. Garretson, University of Michigan.
J. W. Glover, University of Michigan.
E. D. Grant, Michigan College of Mines.
C. L. Herron, Hillsdale College.
T. H. Hildebrandt, University of Michigan.
W. J. Hussey, University of Michigan.
L. C. Karpinski, University of Michigan.
W. W. Küstermann, University of Michigan.
E. A. Lyman, State Normal College.
J. L. Markley, University of Michigan.
G. R. Mirick, University of Michigan.
Alfred Nelson, University of Michigan.
F. N. Notestein, Alma College.
L. C. Plant, Michigan Agricultural College.
R. H. Reece, Michigan Agricultural College.
Wm. Rinck, Calvin College.
T. R. Running, University of Michigan.
E. R. Sleight, Albion College.
G. G. Speeker, Michigan Agricultural College.
C. C. Spooner, Northern Normal School.
W. J. Thome, University of Detroit.
M. O. Tripp, Olivet College.
Mary L. Welton, Union High School, Grand Rapids.
Marion B. White, Michigan State Normal College.
C. B. Williams, Kalamazoo College.
Alexander Ziwet, University of Michigan.

MINNESOTA.

Albert Babbitt, University of Minnesota.
G. N. Bauer, University of Minnesota.
W. O. Beal, University of Minnesota.
Edla G. Berger, College of St. Catherine.
W. E. Brooke, University of Minnesota.
W. H. Bussey, University of Minnesota.
H. H. Dalaker, University of Minnesota.
C. H. Gingrich, Carleton College.
G. W. Hartwell, Hamline University.
A. F. Kovarik, University of Minnesota.
L. E. Lunn, School Superintendent, Heron Lake.
B. L. Newkirk, University of Minnesota.
C. A. V. Peterson, Minneapolis.
W. D. Reeve, University of Minnesota.
R. R. Shumway, University of Minnesota.
H. L. Slobin, University of Minnesota.
J. F. Taylor, Central High School, Duluth.
A. L. Underhill, University of Minnesota.

MISSISSIPPI.

Alfred Hume, University of Mississippi.
J. M. Sharp, Mississippi College.
B. M. Walker, Agricultural and Mechanical College.

MISSOURI.

L. D. Ames, University of Missouri.
Charles Ammerman, McKinley High School, St. Louis.
A. C. Andrews, Manual Training High School, Kansas City.
C. J. Borgmeyer, St. Louis University.
M. S. Brennan, St. Louis.
Dorothy G. Calman, St. Louis.
A. D. Campbell, Washington University.
E. F. Canaday, Columbia.
Byron Cosby, State Normal School, Kirksville.
Otto Dunkel, University of Missouri.
C. A. Epperson, First District Normal School.
Zoe Ferguson, St. Joseph Junior College.
B. F. Finkel, Drury College.
R. R. Fleet, William Jewell College.
G. C. Forsman, Central High School, St. Louis.
Emma M. Gibson, Drury College.
E. R. Hedrick, University of Missouri.
C. G. Hinrichs, Consulting Chemist, St. Louis.
Louise H. Huff, McKinley High School, St. Louis.
Jewell C. Hughes, Columbia.
A. H. Huntington, Central High School, St. Louis.
Byron Ingold, Christian University.
Louis Ingold, University of Missouri.
T. W. Jackson, Fulton High School.
John James, Synodical College.

G. H. Jamison, First District Normal School.
J. R. Jenison, Tarkio College.
B. F. Johnson, State Normal School, Cape Girardeau.
Stella Johnson, Edina High School.
O. D. Kellogg, University of Missouri.
J. M. Kent, Manual Training High School, Kansas City.
Lyda Long, Cleveland High School, St. Louis.
W. A. Luby, Northeast High School, Kansas City.
A. R. Nauer, Mechanical Engineer, St. Louis.
Randolph Patton, Columbia.
J. C. Rayworth, Washington University.
W. H. Roever, Washington University.
W. G. Rowe, Smith Academy Manual Training School.
J. H. Scarborough, State Normal School, Warrensburg.
A. J. Schwartz, Grover Cleveland High School, St. Louis.
J. I. Shannon, St. Louis University.
I. C. Smith, Columbia.
H. P. Stellwagen, Yeatman High School, St. Louis.
F. C. Touton, St. Joseph Junior College.
F. W. Urban, State Normal School, Warrensburg.
C. A. Waldo, Washington University.
J. A. Waldron, Chaminade College.
Eula A. Weeks, Cleveland High School, St. Louis.
R. A. Wells, Park College.
W. D. A. Westfall, University of Missouri.
Rose B. Wood, Hardin College.
W. H. Zeigel, First District Normal School.

MONTANA.

H. F. Calderwood, Glasgow.
E. F. A. Carey, University of Montana.
N. J. Lennes, University of Montana.

NEBRASKA.

J. N. Bennett, Doane College.
Henry Blumberg, University of Nebraska.
W. C. Brenke, University of Nebraska.
G. R. Chatburn, University of Nebraska.
E. W. Davis, University of Nebraska.
R. I. Elliott, State Normal School, Kearney.
H. C. Feemster, York College.
T. J. Fitzpatrick, Bethany.
Ellen H. Frankish, Omaha High School.
J. M. Howie, State Normal School.
Mayme I. Logsdon, Hastings College.
L. E. Pratt, Tecumseh.
W. M. Reeves, Cotner University.
Oscar Schmiedel, Bellevue College.
L. C. Walker, Ceresco.

NEVADA.

Chas. Haseman, University of Nevada.
J. A. Nyswander, University of Nevada.

NEW HAMPSHIRE.

R. D. Beetle, Dartmouth College.
E. G. Bill, Dartmouth College.
C. R. Dines, Dartmouth College.
G. I. Hopkins, Manchester, High School.
L. C. Mathewson, Dartmouth College.
F. C. Moore, New Hampshire College.
F. M. Morgan, Dartmouth College.
C. C. Steck, New Hampshire State College.
H. L. Sweet, Phillips Exeter Academy.
J. W. Young, Dartmouth College.

NEW JERSEY.

E. P. Adams, Princeton University.
A. A. Bennett, Princeton University.
G. A. Bingley, Princeton.
Sister Blanche Marie, College of St. Elizabeth.
J. W. Colliton, Trenton High School.
L. S. Dederick, Princeton University.
Fletcher Durell, Lawrenceville School.
L. P. Eisenhart, Princeton University.
H. B. Fine, Princeton University.
C. O. Gunther, Stevens Institute of Technology.
William Kent, Consulting Engineer, Montclair.
I. E. Kline, Atlantic City High School.
R. W. Lord, Plainfield High School.
L. A. Martin, Jr., Stevens Institute of Technology.
Richard Morris, Rutgers College.
J. A. Nyberg, Princeton.
F. E. Seymour, State Normal School.
C. A. Stanwick, Electrical Engineer, Orange.
H. D. Thompson, Princeton University.
A. A. Titsworth, Rutgers College.
Oswald Veblen, Princeton University.
H. E. Webb, Central High School, Newark.
F. N. Willson, Princeton University.

NEW MEXICO.

W. E. Edington, University of New Mexico.
D. C. Pearson, New Mexico Military Institute.
T. G. Rodgers, New Mexico Normal University.

NEW YORK.

Joseph Allen, College of the City of New York.
Matilda Auerbach, Ethical Culture High School, New York.
D. R. Belcher, Columbia University.
Mabel R. Benway, Bay Ridge High School, Brooklyn.
C. A. Bergstresser, Boys' High School, Brooklyn.
W. J. Berry, Polytechnic Institute, Brooklyn.
Herman Betz, Cornell University.
William Betz, East High School, Rochester.
Harry Birchenough, State College for Teachers.
Joseph Bowden, Adelphi College.
Jessie W. Boyce, New York.
W. E. Breckenridge, Columbia University.
J. A. Brewster, College of the City of New York.
H. S. Brown, Hamilton College.
W. G. Bullard, Syracuse University.
R. W. Burgess, Cornell University.
J. A. C. Callan, Union University.
G. A. Campbell, Research Engineer, Am. Tel. and Telephone Company, New York.
W. M. Carruth, Hamilton College.
W. B. Carver, Cornell University.
Mary E. Caster, New York.

E. B. Chamberlain, The Franklin School, New York.
G. M. Conwell, State College for Teachers.
E. C. Cook, College of the City of New York.
Elizabeth B. Cowley, Vassar College.
E. O. Cox, New York.
Alfred Davis, Horace Mann High School, New York.
G. G. Day, Buffalo.
F. F. Decker, Syracuse University.
H. R. Dougherty, New York Military Academy.
C. H. Douglas, D. C. Heath and Company, New York.
W. P. Durfee, Hobart College.
C. P. Echols, U. S. Military Academy.
J. O. Eckersley, Consulting Engineer, New York.
T. W. Edmondson, New York University.
J. D. Eshleman, University of Rochester.
F. E. Fash, New York.
C. A. Fischer, Columbia University.
T. S. Fiske, Columbia University.
Mrs. Edward Fitch, Clinton.
W. B. Fite, Columbia University.
F. W. Frankland, Actuary, Equitable Life Assurance Society, New York.
W. S. Franklin, New York.
M. G. Gaba, Cornell University.
A. S. Gale, University of Rochester.
D. C. Gillespie, Cornell University.
Matilda Goertz, High School Teacher, New York.
Benjamin Grossbaum, New York.
C. C. Grove, Columbia University.
H. O. Hanson, East Elmhurst, Long Island.
F. M. Hartmann, Cooper Union.
H. E. Hawkes, Columbia University.
G. M. Hayes, College of the City of New York.
Robert Henderson, Actuary, Equitable Life Assurance Society, New York.
A. A. Himwich, Physician, New York.
Blanche Hirsch, Alcuin Preparatory School, New York.
F. C. Hodgdon, Ginn and Company, New York.
Anna M. Howe, Jordan.
W. A. Hurwitz, Cornell University.
S. A. Joffe, Actuary, Mutual Life Insurance Company, New York.
E. A. Johnson, College of the City of New York.
Edward Kasner, Columbia University.
D. F. Kelly, High School Teacher, New York.
E. H. Koch, Jr., High School of Commerce, New York.
F. A. Kristal, Cascadilla School, Ithaca.
Harry Langman, Metropolitan Life Insurance Company, New York.
J. A. Lanigan, Niagara Falls.
Marcia L. Latham, Hunter College.
Louis Lindsey, Syracuse University.
P. H. Linehan, College of the City of New York.
L. L. Locke, Brooklyn Training School for Teachers.
J. V. McKelvey, Cornell University.
James Maclay, Columbia University.
James McMahon, Cornell University.
E. S. Mayer, Cascadilla School, Ithaca.
Mansfield Merriman, New York.
H. B. Mitchell, Columbia University.
E. C. Molina, American Tel. and Telephone Company, New York.
A. H. Norton, Elmira College.
F. W. Owens, Cornell University.
George Paaswell, Civil Engineer, New York.
F. M. Pedersen, College of the City of New York.
H. F. Pfahl, New York.
Maximilian Philip, College of the City of New York.
President, Pi Mu Epsilon Fraternity, Syracuse University.
Arthur Ranum, Cornell University.
H. W. Reddick, Cooper Union.
Emma M. Requa, Hunter College.
F. G. Reynolds, College of the City of New York.
E. D. Roe, Jr., Syracuse University.
E. L. Sanford, St. Stephen's College.
Paul Saurel, College of the City of New York.
C. N. Schmall, Public Schools, New York.
Elmer Schuyler, Bay Ridge High School, Brooklyn.
H. M. Sheffer, College of the City of New York.
L. P. Siceloff, Columbia University.
L. L. Silverman, Cornell University.
Lao G. Simons, Hunter College.
A. W. Smith, Colgate University.
D. E. Smith, Columbia University.
R. R. Smith, with The Macmillan Company, New York.
R. H. Somers, U. S. Military Academy.
Jessie Spearing, New York.
J. J. Tanzola, Cooper Union.
J. M. Taylor, Colgate University.
W. E. Taylor, Syracuse University.
J. S. Thompson, Mutual Life Insurance Company, New York.
A. B. Turner, College of the City of New York.
C. V. Van Anda, with New York Times, New York.
Anna L. Van Benschoten, Wells College.
J. N. Vedder, Union College.
Evelyn Walker, Hunter College.
C. B. Walsh, Ethical Culture High School, New York.
Louisa M. Webster, Hunter College.
A. L. Wechsler, New York.
Mary E. Wells, Vassar College.
Bertha G. Westfall, New York.
E. E. Whitford, College of the City of New York.
G. F. Wilder, Erasmus Hall High School, Brooklyn.
Ruby Willis, Wells College.

NORTH CAROLINA.

T. C. Amick, Elon College.
Helen Barton, Salem College.
H. H. Brinton, Guilford College.
William Cain, University of North Carolina.
J. L. Douglas, Davidson.
J. W. Lasley, Jr., University of North Carolina.
Gertrude O. Mendenhall, State Normal College.
K. B. Patterson, Lenair College.

Virginia Ragsdale, State Normal College.
W. W. Rankin, Jr., University of North Carolina.
M. R. Richardson, Durham.

NORTH DAKOTA.

R. R. Hitchcock, University of North Dakota.

OHIO.

R. B. Allen, Kenyon College.
F. Anderegg, Oberlin College.
W. E. Anderson, Wittenberg College.
G. N. Armstrong, Ohio Wesleyan University.
C. L. Arnold, Ohio State University.
C. B. Austin, Ohio Wesleyan University.
Grace M. Bareis, Ohio State University.
Mrs. W. E. Beckwith, College for Women, Western Reserve University.
R. D. Bohannan, Ohio State University.
Louis Brand, Jr., University of Cincinnati.
J. B. Brandeberry, Toledo University.
Elizabeth F. Burnell, Lake Erie College.
J. M. Cain, Mechanical Engineer, Ashtabula.
W. D. Cairns, Oberlin College.
A. G. Caris, Defiance College.
F. E. Carr, Oberlin College.
G. E. Carscallen, Hiram College.
E. F. Coddington, Ohio State University.
L. M. Coffin, Oberlin College.
R. M. Deming, Case School of Applied Science.
O. L. Dustheimer, Baldwin-Wallace College.
J. B. Faught, Kent State Normal College.
T. M. Focke, Case School of Applied Science.
Harriet E. Glazier, Western College for Women.
M. E. Graber, Heidelberg University.
C. B. Haldeman, Ross.
Harris Hancock, University of Cincinnati.
E. J. Hirschler, Bluffton College.
Adam Hofmann, St. Mary College.
William Hoover, Ohio University.
Miss M. C. Horn, Muskingum College.
Christian Hornung, Heidelberg University.
C. A. Hutchinson, Wittenberg College.
A. L. Jenkins, University of Cincinnati.
R. A. Johnson, Western Reserve University.
W. W. Johnson, Y. M. C. A. Night Schools, Cleveland.
J. H. Kindle, University of Cincinnati.
Emma L. Konantz, Ohio Wesleyan University.
H. W. Kuhn, Ohio State University.
Gertrude I. McCain, Oxford College for Women.
G. W. McCoard, Ohio State University.
Mrs. Eva S. Maglott, Ohio Northern University.
W. V. Metcalf, Oberlin.
F. E. Miller, Otterbein University.
C. N. Moore, University of Cincinnati.
Charlotte Morningstar, Ohio State University.
C. C. Morris, Ohio State University.
J. R. Overman, Bowling Green State Normal College.
R. E. Owen, Forest.
Anna H. Palmié, College for Women, Western Reserve University.
A. D. Pitcher, Western Reserve University.
J. B. Preston, Ohio State University.
S. E. Rasor, Ohio State University.
Hortense Rickard, Ohio State University.
Mary E. Sinclair, Oberlin College.
S. A. Singer, Capital University.
S. E. Slocum, University of Cincinnati.
E. S. Smith, University of Cincinnati.
M. J. Spinks, Bridge Engineer, Wilmington.
J. M. Stetson, Western Reserve University.
K. D. Swartzel, Ohio State University.
C. F. Thomas, Case School of Applied Science.
R. P. Thomas, College of Wooster.
S. E. Urner, Miami University.
A. W. Wallace, Franklin College.
C. J. West, Ohio State University.
Mabel G. Whiting, Antioch College.
F. B. Wiley, Denison University.
D. T. Wilson, Case School of Applied Science.
D. W. Woodard, Wilberforce University.
B. F. Yanney, College of Wooster.
E. I. Yowell, University of Cincinnati.

OKLAHOMA.

E. P. R. Duval, University of Oklahoma.
H. C. Gossard, University of Oklahoma.
Carl Gundersen, Agricultural and Mechanical College.
J. R. Livingston, School of Mines.
P. S. Morgan, Henry Kendall College.
S. W. Reaves, University of Oklahoma.
J. L. Riley, Northeastern State Normal.
W. T. Short, Oklahoma Baptist University.

OREGON.

E. E. De Cou, University of Oregon.
F. L. Griffin, Reed College.
H. H. Ludlow, U. S. Army, Fort Stevens.
A. A. Merriss, Portland.
Emily G. Palmer, Salem High School.
Gilbert Thayer, Engineer, Portland.
E. D. West, Pacific University.

PENNSYLVANIA.

O. P. Akers, Allegheny College.
A. T. G. Apple, Franklin and Marshall College.
C. S. Atchison, Washington and Jefferson College.
A. C. Baird, Fifth Avenue High School, Pittsburgh.
J. A. Bauman, Muhlenberg College.
O. F. H. Bert, Washington and Jefferson College.
F. L. Bishop, University of Pittsburgh.
J. C. Bland, Bridge Engineer, Pittsburgh.
J. F. Burley, Philadelphia.
G. G. Chambers, University of Pennsylvania.
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H. W. Richmond, King's College, Cambridge.

INDIA.

N. P. Pandya, Sojitra High School.

Following is the list of Institutional Members received up to April 24. There being no initiation fee for this class of members, applications may be sent at any time, accompanied by the annual dues of five dollars.

INSTITUTIONAL MEMBERS.

University of St. Francis Xavier, Antigonish, N. S., Canada.
Colorado College, Colorado Springs, Colo.
Colorado School of Mines, Golden, Colo.
Wesleyan University, Middletown, Conn.
University of Georgia, Athens, Ga.
Carthage College, Carthage, Ill.
Armour Institute of Technology, Chicago, Ill.
University of Chicago, Chicago, Ill.
Northwestern University, Evanston, Ill.
Rockford College, Rockford, Ill.
University of Illinois, Urbana, Ill.
Purdue University, LaFayette, Ind.
Manchester College, North Manchester, Ind.
Iowa State College, Ames, Ia.
State University of Iowa, Iowa City, Ia.
University of Kansas, Lawrence, Kans.
Bethel College, Newton, Kans.
University of Louisville, Louisville, Ky.
University of Maine, Orono, Me.
Amherst College, Amherst, Mass.
University of Michigan, Ann Arbor, Mich.
Kalamazoo College, Kalamazoo, Mich.
University of Minnesota, Minneapolis, Minn.
College of St. Catherine, St. Paul, Minn.
Blue Mountain College, Blue Mountain, Miss.
University of Missouri, Columbia, Mo.
Central College, Fayette, Mo.

Washington University, St. Louis, Mo.
University of Montana, Missoula, Mont.
Creighton University, Omaha, Nebr.
Rutgers College, New Brunswick, N. J.
New York State College for Teachers, Albany, N. Y.
University of Buffalo, Buffalo, N. Y.
The College of the City of New York, New York, N. Y.
The Cooper Union, New York, N. Y.
Case School of Applied Science, Cleveland, Ohio.
Western Reserve University, Cleveland, Ohio.
Ohio State University, Columbus, Ohio.
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Kenyon College, Gambier, Ohio.
Oberlin College, Oberlin, Ohio.
Lake Erie College, Painesville, Ohio.
Albright College, Myerstown, Pa.
Drexel Institute, Philadelphia, Pa.
Carnegie Institute of Technology, Pittsburgh, Pa.
Swarthmore College, Swarthmore, Pa.
University of Texas, Austin, Tex.
Southern Methodist University, Dallas, Texas.
Baylor University, Waco, Texas.
Middlebury College, Middlebury, Vt.
University of Virginia, University, Va.
University of Washington, Seattle, Wash.

Summary of Charter Membership.

Individual members whose acceptances have been received	1,028
Individuals elected as signers of the "Call" or participants in the Columbus meeting, from whom no word has been received	18
Institutional members	52
Total	1,098

In addition to the above figures, there are on the MONTHLY subscription list the names of 141 persons who have not as yet become members of the ASSOCIATION, and of 176 libraries not included among the institutional members.

NOTES AND NEWS.

EDITED BY D. A. ROTHROCK, Indiana University.

Dr. DUNHAM JACKSON has been promoted to an assistant professorship of mathematics at Harvard University.

Mr. C. GARLOUGH has been appointed instructor in mathematics at Wheaton College, Illinois.

The death is announced of Professor J. W. RICHARD DEDEKIND, of the technical school of Brunswick, Germany.

In *Science* of March 3, Professor E. V. HUNTINGTON has a short and interesting discussion on "The fundamental equation of mechanics."

Miss S. F. RICHARDSON, assistant professor of mathematics in Vassar College, died on Feb. 2, 1916. Miss Richardson was a graduate of Vassar, and for thirty years has been a member of the teaching staff.

Professor ARNOLD EMCH, of the University of Illinois, has an interesting non-technical article in *The Scientific Monthly*, March, 1916, on "The representation of large numbers and infinite processes."

At Smith College, Miss HARRIET R. COBB has been promoted to a professorship of mathematics, Miss PAULINE SPERRY has been appointed assistant professor of mathematics, and Dr. MARY B. HOPKINS has been promoted to an associate professorship of astronomy.

In the *Journal of Educational Psychology* for February, Professor H. L. RIETZ has a statistical paper on "The correlation of marks of students in mathematics and in law."

A paper prepared by President Emeritus CHARLES W. ELIOT for the General Education Board on "Changes needed in American secondary education" appeared in *School and Society*, March 16.

A Plane Geometry by C. G. PALMER, associate professor of mathematics in the Armour Institute, and D. P. TAYLOR of the Oak Park High School, Chicago, has appeared from the press of Scott, Forsman & Co.

The following Cambridge tracts in mathematics are announced to appear soon: "The definite integral," by E. W. HOBSON; "An introduction to the theory of attraction," by T. J. BROMWICH; "Pascal's hexagon," by H. W. RICHMOND; "Lemniscate functions," by G. B. MATHEWS; "Chapters on algebraic geometry," and "Integrals of algebraic functions," by T. H. BAKER.

The publishing house of Lyons and Carnahan, Chicago, have just issued a text on Solid Geometry for secondary schools by J. H. WILLIAMS, teacher of mathematics in the Urbana, Ohio, High School, and K. P. WILLIAMS, assistant professor of mathematics at Indiana University. This book is the sequel to the authors' Plane Geometry, which appeared some months ago.

A second edition of E. T. WHITTAKER'S "A course of modern analysis" has appeared from the Cambridge University Press. The same publishers also announce "Euclid's book on division of figures" with a restoration based on Woepcke's text, by Professor R. C. ARCHIBALD, of Brown University. This book of 188 pages is a restoration of Euclid's ninth book, which has not heretofore been attempted in English.

The January number of *Proceedings of the London Mathematical Society* contains the following papers: "On Dirichlet's divisor problem," by G. H. HARDY; "On periodic irrotational waves at the surface of deep water," by W. BURNSIDE; "Proof of the complementary theorem," by J. C. FIELDS; "On integrals and derivatives with respect to a function," by W. H. YOUNG; "The effect on the tides of variation in the depth of the sea," by G. R. GOLDSBROUGH; "The second theorem of consistency for summable series," by G. H. HARDY.

The American Journal of Mathematics for January contains the following papers: "The oscillations of an orthogonal set," by O. D. KELLOGG; "On some properties of the medians of closed continuous curves formed by analytic arcs," by ARNOLD EMCH; "Theorems on groups of isomorphisms of certain groups," by L. C. MATHEWSON; "Self-projective rational sextics," by R. M. WINGER; "On linear difference and differential equations," by C. E. LOVE; "The uniform motion of a sphere through a viscous liquid," by R. W. BURGESS; "Note on the theory of optical images," by G. STEIN.

A reprint from the Napier Tercentenary memorial volume, published by the Royal Society of Edinburgh, contains a short article by the veteran mathematician, ARTEMAS MARTIN, LL.D., of Washington, on "A method of finding without the use of tables the number corresponding to a given natural logarithm." The memorial volume also contains an address by Professor D. E. SMITH on "The law of exponents in the works of the sixteenth century," delivered at the Napier Tercentenary.

The Mathematics Teacher, September, 1915, contains an instructive address by Professor A. D. PITCHER, of Dartmouth College, delivered before the Association of Mathematics Teachers of New England on the "Reorganization of the mathematical curriculum in the secondary schools." Professor Pitcher's conclusions indicate that: (1) No serious change should be made in the curriculum of the secondary school until after careful consideration based upon skilled experimentation; (2) much of the criticism against the curriculum will disappear with the increasingly careful and thorough preparation of teachers; (3) we should let our conservatism operate only to prevent the adoption of reform measures without due experimentation and experimental evaluation; (4) the curriculum at present is good; however, the following minor changes are suggested: (*a*) omission of difficult technique of algebra; (*b*) intuitional approach to geometry; (*c*) introduction of the function concept early in algebra; (*d*) introduction of the elements of trigonometry.

The Mathematics Club of Mt. Holyoke College is an organization of junior and senior students in mathematics, and the members of the mathematical teaching staff, one of whom is on the executive committee. The purpose of the club as stated in its constitution is "to present to its members a broader view of mathematics." Attendance and membership are voluntary. Regular meetings are held once a month, a number during the year being social gatherings; the remaining meetings consist of regular programs for presentation of papers and reports by student and faculty members, upon various historical and non-technical features of mathematics.

Summer courses in graduate and undergraduate mathematics are offered by a large number of colleges and universities. Below are enumerated those courses already announced, so far as they have come to the notice of the editor.

DARTMOUTH COLLEGE: Summer session, July 16 to August 16. By Professor J. W. YOUNG: The reorganization of secondary school mathematics.—By Professor E. G. BILL: Plane analytic geometry; Projective geometry.

INDIANA UNIVERSITY: Summer session, June 15 to August 11. By Professor S. C. DAVISSON: Differential geometry, 5 hrs.; Projective geometry, 3 hrs.—By Professor D. A. ROTHROCK: Fourier series, 3 hrs.; Differential equations, 5 hrs.—By Professor U. S. HANNA: Solid analytic geometry, 5 hrs.; Elementary calculus, 5 hrs.—By Professor K. P. WILLIAMS: Advanced calculus, 5 hrs. Courses in college algebra, trigonometry, and analytic geometry will also be given.

UNIVERSITY OF MICHIGAN: Summer session, July 30 to August 25. By Professor W. W. BEMAN: Differential equations; Teacher's course in algebra and geometry.—By Professor J. L. MARKLEY: Functions of a complex variable; Advanced analytics; Advanced algebra.—By Professor W. B. FORD: Advanced calculus; Theory of potential; Infinite series and products.—By Professor L. C. KARPINSKI: History of mathematics.—By Dr. C. H. FORSYTH: Mathematical theory of finance, insurance and statistics.—By Dr. V. C. POOR: Calculus.—By Mr. COE: Calculus; Mechanics. Courses in elementary algebra, trigonometry and analytics are also announced.

UNIVERSITY OF WISCONSIN: Summer session, June 26 to August 14. By Professor E. B. SKINNER: Differential geometry; Linear substitutions; Complex numbers.—By Professor H. W. MARCH: Theoretical mechanics; Infinite series.—By Professor H. C. WOLFF: Probabilities and statistics; Differential calculus.—By Professor W. W. HART: Analytic geometry.—By Mr. TAYLOR: Elementary analysis for engineering students.—By Mr. PAINE: Integral calculus.—By Mr. SIMPSON: Mathematical theory of investment for students of commerce. The usual elementary courses in geometry, algebra and trigonometry are also provided.

THE UNIVERSITY OF CHICAGO: Summer session, June 17 to September 1. By Professor E. H. MOORE: Theory of limits (first term); Integral equations in general analysis (first term).—By Professor G. A. BLISS: Integral calculus; Theory of functions of a real variable.—By Professor L. E. DICKSON: Substitution groups and algebraic equations; Solution of numerical equations (first term); Determinants and symmetric functions (second term).—By Professor H. E. SLAUGHT: Differential calculus; Elliptic integrals.—By Professor F. R. MOULTON: Theory of infinite series (second term); Theory of functions of infinitely many variables (second term).—By Professor J. W. A. YOUNG: Selected topics in mathematics.—By Professor W. D. MACMILLAN: Descriptive astronomy; Introduction to celestial mechanics.—By Professor ARTHUR RANUM: Metric differential geometry.—By Dr. C. R. DINES: Differential equations. Courses in trigonometry, college algebra, and plane analytic geometry are announced; and also courses in reading and research for advanced students. The mathematical club meets each week for the reports of research and the consideration of questions of teaching, alternating these topics.

IMPORTANT NOTICE TO FORMER MONTHLY SUBSCRIBERS.

Henceforth, the subscription price of the MONTHLY will be *three dollars net to all non-members of the* ASSOCIATION.

The following adjustments are proposed for those subscribers who failed to apply for membership in the ASSOCIATION before April first.

(1) Any person who had already paid $2.00 on the old basis for the year 1916 is asked to send *one dollar additional* to complete the subscription. If such a person desires also to become a member of the ASSOCIATION, an initation fee of $2.00 must now be paid.

(2) Those whose subscriptions for 1916 are entirely unpaid will now pay $3.00 for the subscription or $5.00 for membership in the ASSOCIATION, including the subscription.

In the case of subscriptions under (1) or (2) which expire *before* the end of 1916, please add *thirty cents extra* for each copy needed to complete the year. *Hereafter all subscriptions will date from January of each year.*

(3) An institution in which the Calculus is taught may become an *institutional member* of the ASSOCIATION by the payment of *five dollars* annually, which will entitle the library to receive *two copies of the* MONTHLY and the institution to send a voting delegate to all meetings of the ASSOCIATION. Institutions in which the Calculus is taught, whose libraries have already renewed their subscriptions for 1916, are asked to send *three dollars additional* and thus become institutional members of the ASSOCIATION.

Other institutions, and those not wishing to become institutional members, whose library subscriptions have already been renewed for 1916, are asked to send *one dollar additional* to complete the advanced price of the MONTHLY. No further subscriptions will be received at the old rate of two dollars, *and no discount from the advanced rate of three dollars will be allowed on subscriptions under any circumstances.*

(4) The obligations of the MONTHLY for 1916 will, of course, be fulfilled on the former basis in the case of any individual or institution whose subscription has already been paid, and who may decline to make the adjustment on the new basis, but it is hoped that all will comply, since even the advanced rate is only the actual cost of producing the journal.

(5) Please note that all subscriptions to the MONTHLY and dues in the ASSOCIATION are to be paid to the SECRETARY-TREASURER, Professor W. D. CAIRNS, 55 East Lorain St., Oberlin, Ohio.

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EDITED BY

H. E. SLAUGHT W. H. BUSSEY R. D. CARMICHAEL

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